Problem 2:

The equation of an ellipse in the cartesian co-ordinate system \((x,y)\) is:

\[
\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1.
\]  

(1)

Taking derivative w.r.t. \(x\) on both sides of Equation 1,

\[
\frac{2x}{a^2} + \frac{2y}{b^2} \frac{dy}{dx} = 0.
\]  

(2)

Equation 2 and its further derivative w.r.t. \(x\) yield

\[
\frac{dy}{dx} = -\frac{b^2}{a^2} \frac{x}{y}
\]  

(3)

\[
\frac{d^2y}{dx^2} = -\frac{b^4}{a^2 y^3}
\]

(4)

Hence,

\[
\rho = \left. \frac{1 + (dy/dx)^2} {d^2y/dx^2} \right|_{x=a, y=0}
\]  

\[
= \left. \frac{1 + \left( -\frac{b^2 x}{a^2 y} \right)^2} {\frac{-b^4}{a^2 y^3}} \right|_{x=a, y=0}
\]

\[
= \left. \frac{a^4 y^2 + b^4 x^2} {a^4 b^4} \right|_{x=a, y=0}
\]

\[
= \frac{b^2}{a}
\]

(5)

which gives

\[
b = \sqrt{\rho a} \quad (b \text{ is positive})
\]

(6)

Therefore,

\[
\sigma \mid_{x=a, y=0} = \sigma^\infty \left[ 1 + 2 \frac{a}{b} \right] = \sigma^\infty \left[ 1 + 2 \frac{a}{\sqrt{\rho}} \right]
\]

(7)