This assignment concerns the use of frequency response in systems analysis and design. You must use Matlab for this assignment. Before stating the precise problems that you must complete, we give below the basic matlab features that can be used to generate the frequency response of the system.

**Plotting the frequency response:** First thing you need to set up is the vector of frequency values at which the frequency response has to be computed. The quickest way to do this is to use the matlab command

\[ w = a:b:c; \]

This sets up a vector whose first element is a, the second element a+b, the third a+2b, etc. with the last element c, provided of course c-a is an integer multiple of b. Thus, the command

\[ w = 0:.1:1 \]

gives the vector

\[ w = [0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0]. \]

Now suppose the frequency response of the system is

\[
H(\omega) = \frac{b_1(j\omega)^m + b_2(j\omega)^{m-1} + \ldots + b_{m+1}}{(j\omega)^n + a_1(j\omega)^{n-1} + \ldots + a_n}. \tag{1}
\]

Then define:

\[ a = [1, a_1, a_2, \ldots, a_n] \tag{2} \]

and

\[ b = [b_1, b_2, \ldots, b_{m+1}]. \tag{3} \]

Then the matlab commands

\[
h = \text{freqs}(b,a,w);
mag = \text{abs}(h);
\text{phase} = \text{angle}(h) \times 180/\pi;
\]

puts the magnitude and phase values in the vectors mag and phase respectively. Note the elements of phase are in degrees as opposed to radians. For example, for the system having frequency response

\[
H(\omega) = \frac{j\omega + 2}{(j\omega)^3 + 3(j\omega)^2 + 3j\omega + 1}. \tag{4}
\]
the commands
w=0:.01:1;
a=[1,3,3,1];
b=[1,2];
h = freqs(b,a,w);
mag = abs(h);
phase = angle(h)*180/pi;
will create the vectors

\[
\text{mag} = \begin{bmatrix}
|H(0)| \\
|H(0.01)| \\
|H(0.02)| \\
\vdots \\
|H(1)|
\end{bmatrix}
\]

(5)

and

\[
\text{phase} = \begin{bmatrix}
\text{angle}(H(0)) \\
\text{angle}(H(0.01)) \\
\text{angle}(H(0.02)) \\
\vdots \\
\text{angle}(H(1))
\end{bmatrix}
\]

(6)

Then the commands
plot(w,mag);
plot(w,phase);
give the magnitude and phase response plots.

**The Theory:** Recall that if a system has frequency response \( H(j\omega) = |H(j\omega)|e^{j\theta(\omega)} \) then the output due generated by the input \( C\cos(\omega_0 t) \) is

\[
C|H(j\omega_0)| \cos(\omega_0 t + \theta(\omega_0))
\]

**The Problem:** Consider the signal

\[
x(t) = \sin(t) + \cos(2t).
\]
Plot $x(t)$ for $t$ in $[0, 10]$ Goal is to design butterworth filters of degree 5, 10, 15, 20 and 25, so that the higher frequency component is filtered out. Design suitable filters of these orders. In each case plot its magnitude and phase response. In each case theoretically determine the output and plot it. Comment on the efficacy with which the design goal was met.