Adaptive Probability Analysis Using Performance Measure Approach

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ABSTRACT

This paper presents an adaptive probability analysis to identify the effect of input uncertainty. It was found in the literatures that existing probability analysis methods do not perform well in terms of numerical efficiency, accuracy, and/or robustness. These difficulties can be resolved by utilizing the performance measure approach (PMA) and accurately approximating the most probable point (MPP) locus that provides initial search points for the probability analysis process. The MPP locus approximation is studied by employing a least squares (LS) and a moving least squares (MLS) methods. The proposed probability analysis method adaptively adds more probability levels by using a posteriori-error estimator, such that an appropriate number of probability levels is adaptively decided while constructing probability function of output performance functions. The hybrid mean value (HMV) method is numerically implemented in the proposed adaptive probability analysis using performance measure approach (PMA).

INTRODUCTION

The fact that engineering system design decisions need to be made under various uncertainties has been getting more attention in recent years. As a result, it is widely recognized the need of methods and tools that are able to identify the effect of input uncertainties by identifying the distribution functions of output performance functions. Difficulties in obtaining accurate distribution functions of system responses have highlighted the need for some probability analysis methods: a sampling method, a moment matching method, and a reliability index method.

As a sampling method, Monte Carlo simulation (MCS) [1] has been extensively used in engineering probability analysis, due to its generality and simplicity. However, an extremely large sample size is required for MCS to obtain the probability function of output performance function. Therefore, MCS is seldom used if analysis of the engineering system requires intensive computation, such as with finite element analysis (FEA) and structural durability analysis [2]. Alternatively, the moment matching method is simple and inexpensive to use, but not appropriate when the system is exposed to significant uncertainties. Subsequently, the second-order moment matching method is used to identify the probability bounds rather than the probability distribution [3]. Furthermore, low order moments do not provide the shape of the resulting probability distribution. Recently, the reliability index method [4,5] has become more popular with the advent of the performance measure approach (PMA) employing a family of mean-based methods: the mean value, advanced mean value (AMV) [6], AMV+ [7], and hybrid mean value (HMV) methods [8]. A novel approach to probability analysis was proposed by approximating an MPP locus [9]. Nonetheless, this methodology could be still expensive, since MPP locus is approximated by an extrapolate LS method. Hence, current probability analysis methods are not only ineffective in reliability analysis (i.e., MPP search), but also unable to properly set the number of probability levels, since a set of probability levels must be predetermined without knowing the degree of nonlinearity of the performance function [9,10].

To overcome these limitations, in this paper, the reliability analysis is carried out using the PMA with HMV method. Moreover, the MPP locus is first approximated and reliability analyses start from the corresponding initial search points on the approximated MPP locus at each probability level, rather than from the origin in the independent and standardized normal $U$-space. The number of probability levels is adaptively increased until all initial search points from the approximated MPP locus are close to the corresponding MPPs, depending on the nonlinearity of the probabilistic performance function. Consequently, this paper proposes a new efficient probability analysis method by approximating the MPP locus for the initial search point, and adaptively constructing accurate distribution functions of performance functions. The proposed method is demonstrated using several examples and comparisons are made with some existing probability analysis methods.
RELIABILITY METHODS FOR PROBABILITY
ANALYSIS

In a reliability index method using PMA, a reliability analysis becomes an optimization problem with one equality constraint, formulated as \([8,11,12]\)

\[
\begin{align*}
\text{minimize} & \quad G(U) \\
\text{subject to} & \quad \|U\| = \beta_i
\end{align*}
\]

(1)

where \(G(U)\) is the performance function in \(U\)-space and \(\beta_i\) is the target reliability.

For PMA, although the advanced mean value (AMV) method behaves well for convex performance functions, it was found to have some numerical shortcomings, such as slow convergence, or even divergence, when applied to concave performance functions. To overcome this difficulty, the conjugate mean value (CMV) method was proposed in Refs. \(8\) and \(12\). However, even though the CMV method always converges, it is inefficient for convex functions. Consequently, the hybrid mean value (HMV) method was proposed to attain both stability and efficiency in the MPP search for PMA \([8,12]\).

Advanced Mean Value First-Order Method

Formulation of the first-order AMV method begins with the mean value method, defined as

\[
u^*_\text{MV} = \beta_i n(0)
\]

where \(n(0) = -\frac{\nabla X G(\mu)}{\|\nabla X G(\mu)\|} = -\frac{\nabla U G(0)}{\|\nabla U G(0)\|}
\]

(2)

That is, to minimize the performance function \(G(U)\) (i.e. the cost function in Eq. (1)), the normalized steepest descent direction \(n(0)\) is defined at the mean value. The AMV method iteratively updates the direction vector of the steepest descent method at the probable point \(u^{(k)}\) \(\text{AMV}\), initially obtained using the mean value method. Thus, the AMV method can be formulated as

\[
u^{(i)} = u^{(i)}\text{MV} , \quad u^{(k+1)} = \beta_i n(u^{(k)}\text{AMV})
\]

(3)

where

\[
\begin{align*}
n(u^{(k)}\text{AMV}) = -\frac{\nabla U G(u^{(k)}\text{AMV})}{\|\nabla U G(u^{(k)}\text{AMV})\|}
\end{align*}
\]

(4)

As will be shown, this method exhibits instability and inefficiency in solving concave functions since the direction is only updated using the current MPP.

Conjugate Mean Value First-Order Method

When applied to a concave function, the AMV method tends to be slow in convergence or to be divergent due to lack of utilization of updated information during the iterative reliability analysis. This type of difficulty can be overcome by using both current and previous iteration information as applied in the CMV method. The new search direction is obtained by combining \(n(u^{(k-2)}\text{CMV})\), \(n(u^{(k-1)}\text{CMV})\), and \(n(u^{(k)}\text{CMV})\) with an equal weight, such that it is directed towards the diagonal of the three consecutive steepest descent directions. That is,

\[
u^{(0)} = 0 , \quad u^{(i)} = u^{(i)}\text{AMV} , \quad u^{(2)} = u^{(2)}\text{AMV},
\]

\[
u^{(k+1)} = \beta_i \frac{n(u^{(k)}\text{CMV}) + n(u^{(k-1)}\text{CMV}) + n(u^{(k-2)}\text{CMV})}{\|n(u^{(k)}\text{CMV}) + n(u^{(k-1)}\text{CMV}) + n(u^{(k-2)}\text{CMV})\|} \quad \text{for} \quad k \geq 2
\]

(5)

where

\[
n(u^{(k)}\text{CMV}) = -\frac{\nabla U G(u^{(k)}\text{CMV})}{\|\nabla U G(u^{(k)}\text{CMV})\|}
\]

(6)

Consequently, the conjugate steepest descent direction significantly improves the rate of convergence as well as the stability, as compared to the AMV method for the concave performance function. However, as seen in the literature \([8,12]\), the CMV method is not as efficient as the AMV method for the convex function.

Hybrid Mean Value First-Order Method

To select the appropriate MPP search method between AMV and CMV, the type of performance measure must first be identified. In this paper, a function type criterion employs the steepest descent directions at three consecutive iterations, as follows:

\[
s^{(k+1)} = (n^{(k+1)} - n^{(k)}) \cdot (n^{(k)} - n^{(k-1)}) \quad \text{sign}(s^{(k+1)}) > 0 : \text{Convex type at } u^{(k+1)}\text{HMV} \text{ w.r.t. design } d
\]

\[
\leq 0 : \text{Concave type at } u^{(k+1)}\text{HMV} \text{ w.r.t. design } d
\]

where \(s^{(k+1)}\) is the criterion for the performance function type at \(k+1\)th step and \(n^{(k)}\) is the steepest descent direction of a performance function at MPP \(u^{(k)}\text{HMV}\) at \(k\)th iteration. Once the performance function type is identified, either AMV or CMV is adaptively selected for the MPP search. The numerical procedure of the HMM method is therefore summarized as following.

Step 1. Set the iteration counter to \(k=0\). Select the convergence parameter \(\varepsilon\). Compute the steepest descent direction of the performance function in \(U\)-space,
\[ n(u_{HMV}^{(0)}) = -\frac{\nabla_U G(u_{HMV}^{(0)})}{\|\nabla_U G(u_{HMV}^{(0)})\|} \quad \text{where} \quad u_{HMV}^{(0)} = 0 \]

Step 2. If the performance function type is convex or \( k < 3 \), carry out the MPP search using the AMV method (note that Step 2 of the AMV method is the same as Step 2 of the HMV method when \( k < 3 \)) as

\[ u_{HMV}^{(k+1)} = \beta \cdot n(u_{HMV}^{(k)}) \]

If the performance function is concave and \( k \geq 3 \), carry out MPP search using the CMV method, as

\[ u_{CMV}^{(k+1)} = \beta \cdot \frac{n(u_{CMV}^{(k)}) + n(u_{CMV}^{(k-1)}) + n(u_{CMV}^{(k-2)})}{n(u_{CMV}^{(k)}) + n(u_{CMV}^{(k-1)}) + n(u_{CMV}^{(k-2)})} \]

where

\[ n(u_{CMV}^{(k)}) = -\frac{\nabla_U G(u_{CMV}^{(k)})}{\|\nabla_U G(u_{CMV}^{(k)})\|} \]

Step 3. Calculate the performance \( G(u_{CMV}^{(k+1)}) \) and the reliability index \( \beta^{(k+1)} \) at the new MPP \( u_{CMV}^{(k+1)} \).

Check to see if

\[ \max \left( \left| \beta^{(k+1)} - \beta \right|, \left| \Delta G_{rel}^{(k+1)} \right|, \left| \Delta G_{abs}^{(k+1)} \right| \right) \leq \epsilon \]

where

\[ \Delta G_{rel}^{(k+1)} = \frac{G(u_{CMV}^{(k+1)}) - G(u_{CMV}^{(k)})}{G(u_{CMV}^{(k+1)})} \] and

\[ \Delta G_{abs}^{(k+1)} = G(u_{CMV}^{(k+1)}) - G(u_{CMV}^{(k)}) \]

If the convergence criteria hold, then stop. Otherwise, go to Step 4.

Step 4. Compute the gradient \( \nabla_U G(u_{CMV}^{(k+1)}) \) of the performance function and check the criterion \( \zeta^{(k+1)} \) for performance function type. Set \( k = k + 1 \) and return to Step 2.

**ADAPTIVE PROBABILITY ANALYSIS**

Utilizing the previously explained HMV method, an adaptive probability analysis method is proposed in this paper to effectively estimate the probability distribution of the performance function. For this, two key ideas are proposed to be integrated to develop the adaptive probability analysis. The first idea is using an MPP locus interpolation rather than extrapolation and the second idea is using an adaptive method to choose probability levels properly. The MPP locus interpolation is studied by comparing LS and MLS methods. The adaptive method uses a posteriori-error estimator, defined as the difference between the initial search point on the approximate MPP locus and the MPP point obtained using the HMV method. Using a posteriori error estimator, the probability level is adaptively set to refine the MPP locus. Thus, the adaptive method provides a close-looped probability analysis, as shown in Figure 1, unlike the usual open-looped probability analysis methods. Detail discussions on two new ideas are presented in next two subsections.

![Figure 1. Flow Chart of Adaptive Probability Analysis](image-url)

**MPP Locus Approximation**

Existing probability analysis methods do not properly set the number of probability levels, since a set of probability levels must be predetermined without knowledge of the degree of nonlinearity of the performance function. Furthermore, reliability analyses at one probability level are independently performed from another probability level. Therefore, much of the valuable information generated during probability analysis at one probability level is ignored at different probability levels. These shortcomings of existing probability analysis methods can be overcome by accurately approximating the MPP locus, which provides initial search points close to MPPs for probability analysis. The MPP locus can be approximated by identifying the relationship between the MPPs and the corresponding probability levels \( U_j = U_j(\beta) \) for \( j=1,...,N_{RV} \). MPP locus approximation is studied by comparing the LS and MLS methods.

1. **Using Least Squares (LS) Method**

MPP locus approximation using the LS method can be formulated as
\[ \bar{U}_j(\beta) = \sum_{i=1}^{NB} h_i(\beta)a_{ij} = \mathbf{h}^T \mathbf{a}_j, \quad \text{for } j = 1, \ldots, NRV \] (8)

where \( NB \) is the number of basis monomials, \( NRV \) is the number of random parameters, \( \mathbf{a}_j \) is the coefficient vector for the \( j \)th random parameter, and \( \mathbf{h} \) is the basis vector. Mutually independent monomials are used as basis functions. In this study, a quadratic polynomial basis is used to approximate the MPP locus.

To obtain the coefficient vector the residual functional \( E_{LS} \) can be defined as

\[ E_{LS}^j = \sum_{i=1}^{NPL} (\bar{U}_j(\beta_i) - U_j(\beta_i))^2 \]

\[ = \sum_{i=1}^{NPL} (\mathbf{h}^T \mathbf{a}_j - U_j(\beta_i))^2 \]

\[ = (\mathbf{H}_j - \mathbf{U}_j)^T (\mathbf{H}_j - \mathbf{U}_j) \]

where

\[ \mathbf{H}_j = \begin{bmatrix} h_1(\beta_1) & \cdots & h_{NB}(\beta_1) \\ \vdots & \ddots & \vdots \\ h_1(\beta_{NPL}) & \cdots & h_{NB}(\beta_{NPL}) \end{bmatrix}, \quad \mathbf{U}_j = \begin{bmatrix} U_j(\beta_1) & \cdots & U_j(\beta_{NPL}) \end{bmatrix}, \]

and \( NPL \) is the number of probability levels.

Differentiation of \( E_{LS}^j \) with respect to the coefficients \( \mathbf{a}_j \) gives the coefficient vector for the \( j \)th random parameter as

\[ \mathbf{a}_j = (\mathbf{H}_j^T \mathbf{H}_j)^{-1} \mathbf{H}_j^T \mathbf{U}_j, \quad \text{for } j = 1, \ldots, NRV \] (11)

The MPP locus approximation using the LS method can then be expressed as

\[ \bar{U}_j(\beta) = \mathbf{h}^T (\mathbf{H}_j^T \mathbf{H}_j)^{-1} \mathbf{H}_j^T \mathbf{U}_j, \quad \text{for } j = 1, \ldots, NRV \] (12)

2. Using Moving Least Squares (MLS) Method

The MPP locus approximation using the MLS method is formulated as

\[ \bar{U}_j(\beta) = \sum_{i=1}^{NB} h_i(\beta)a_{ij}(\beta) \]

\[ = \mathbf{h}^T(\beta) \mathbf{a}_j(\beta), \quad \text{for } j = 1, \ldots, NRV \] (13)

where \( \mathbf{a}_j(\beta) = [a_{1j}(\beta), a_{2j}(\beta), \ldots, a_{NBj}(\beta)]^T \) is the \( j \)th coefficient vector that are a function of reliability \( \beta \), and \( \mathbf{h} \) is the basis vector. Mutually independent monomials are used as basis functions. In this study, a quadratic polynomial basis is used to approximate the MPP locus.

To obtain the coefficient vector the residual functional \( E_{MLS} \) can be defined as

\[ E_{MLS}^j = \sum_{i=1}^{NPL} w(\beta_i - \beta_j)(\bar{U}_j(\beta_i) - U_j(\beta_i))^2 \]

\[ = \sum_{i=1}^{NPL} w(\beta_i - \beta_j)(\mathbf{h}_j^T \mathbf{a}_j - U_j(\beta_i))^2 \]

\[ = [\mathbf{H}_j(\beta) - \mathbf{U}_j]^T \mathbf{W}(\beta)[\mathbf{H}_j(\beta) - \mathbf{U}_j] \]

where

\[ \mathbf{H}_j = \begin{bmatrix} h_1(\beta_1) & \cdots & h_{NB}(\beta_1) \\ \vdots & \ddots & \vdots \\ h_1(\beta_{NPL}) & \cdots & h_{NB}(\beta_{NPL}) \end{bmatrix}, \quad \mathbf{U}_j = \begin{bmatrix} U_j(\beta_1) & \cdots & U_j(\beta_{NPL}) \end{bmatrix}, \]

\[ \mathbf{W}(\beta) = \begin{bmatrix} w(\beta - \beta_1) & 0 & \cdots & 0 \\ 0 & w(\beta - \beta_2) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & w(\beta - \beta_{NPL}) \end{bmatrix} \]

where \( NPL \) is the number of probability levels.

Differentiation of \( E_{MLS}^j \) with respect to the coefficients \( \mathbf{a}_j \) gives the coefficient vector for the \( j \)th random parameter as

\[ \mathbf{a}_j(\beta) = \mathbf{M}^{-1}(\beta) \mathbf{B}(\beta) \mathbf{U}_j, \quad j = 1, \ldots, NRV \] (16)

where \( \mathbf{M}(\beta) \) is referred to as the moment matrix and is given by

\[ \mathbf{M}(\beta) = \mathbf{H}_j^T \mathbf{W}(\beta) \mathbf{H}_j \quad \text{and} \quad \mathbf{B}(\beta) = \mathbf{H}_j^T \mathbf{W}(\beta) \] (17)

The MPP locus approximation using the MLS method is then expressed as

\[ \bar{U}_j(\beta) = \mathbf{h}^T(\beta) \mathbf{M}^{-1}(\beta) \mathbf{B}(\beta) \mathbf{U}_j, \quad j = 1, \ldots, NRV \] (18)

Adaptive Set of Probability Levels

Using either LS or MLS method, the approximate MPP locus is refined, as more probability levels are included. When adding more probability levels to the current set of probability levels, an adaptive method is applied by using a posteriori error estimator, which is defined as

\[ \]
\[ \varepsilon_1 = \begin{cases} \| \hat{u}^0 - u^* \| & \text{for } \beta_i > 1 \\ \| \hat{u}^0 - u^* \| / \beta_i & \text{for } \beta_i \leq 1 \end{cases} \]

and

\[ \varepsilon_2 = \begin{cases} |G(\hat{u}^0) - G(u^*)| & \text{for } |G(u^*)| > 1 \\ |G(\hat{u}^0) - G(u^*)| / |G(u^*)| & \text{for } |G(u^*)| \leq 1 \end{cases} \]

where \( \hat{u}^0 \) is an initial search point, \( u^* \) is the MPP for \( \beta_i \), \( \beta_i \) is the current probability level, and \( G \) is the performance function.

In the proposed probability analysis, a posteriori-error estimation on approximated MPP locus enables to perform the adaptive scheme to decide the number of probability levels appropriately. Subsequently, the MPP locus is iteratively refined, by adding more probability levels. To add probability levels, the adaptive bisection method is used, as shown in Fig. 2. First, a reliability analysis is carried out to find MPP at the highest probability level \( \beta_H \) that is predetermined. Then, the MPP locus is approximated based on the information at two probability levels, 0 and \( \beta_H \). Using the bisection method, the next probability level, \( 1/2 \beta_H \), is selected and the corresponding reliability analysis starting from the initial search point on the approximate MPP locus is carried out to find MPP at this probability level. Then, a posteriori error estimator defined in Eq. (19) is used to check the convergence. If a posteriori error is small, the adaptive process is completed. Otherwise, the above procedure that refines the MPP locus and adaptively adds more probability levels continues, until all initial search points on the approximate MPP locus are very close to the corresponding MPPs. For example, the 3rd refined MPP locus has 5 probability levels. If a posteriori-error estimator is satisfied at 0, \( 1/2 \beta_H \), and \( 1/4 \beta_H \), only 5/8\( \beta_H \) and \( 7/8 \beta_H \) are adaptively added on the 4th MPP locus, as shown in Fig. 2.

### Numerical Procedure of Adaptive Probability Analysis

A numerical procedure for the proposed probability analysis is presented in the following, where numerical steps are applied to two probability regions: more than and less than 50%.

**Step 1.** Set the probability layer counter to \( k = 0 \) and let the highest probability level be \( \beta_H \). Select the convergence parameter \( \varepsilon_{\text{REL}} \) for the reliability analysis, and \( \varepsilon_{\text{MPP}} \) and \( \varepsilon_G \) for MPP approximation. Perform a reliability analysis at \( \beta_H \).

**Step 2.** Select probability levels, where the initial search point is not close to MPP, i.e., \( \varepsilon_G > \varepsilon_{\text{MPP}} \) and \( \varepsilon_G > \varepsilon_G \). If the convergence at all probability levels is achieved, then stop. Otherwise, go to Step 2.

**Step 3.** Obtain initial search points on approximate MPP locus at selected probability levels.

**Step 4.** Perform reliability analyses starting from the initial search points at selected probability levels.

**Step 5.** Check to see if a posteriori-error estimator satisfies the error criteria, i.e., \( \varepsilon_G > \varepsilon_{\text{MPP}} \) and \( \varepsilon_G > \varepsilon_G \). If the convergence at all probability levels is achieved, then stop. Otherwise, go to Step 2.

### RESULTS AND DISCUSSIONS

#### Example 1: Side Impact Crashworthiness for MPP Locus Approximation

For this example, a large-scale application of vehicle side impact is employed, as illustrated in Fig. 3. The system model includes a full-vehicle FE structural model, an FE side impact dummy model, and an FE deformable side impact barrier model. The system model consists of 85,941 shell elements and 96,122 nodes. In the FE simulation of the side impact event, the barrier has an initial velocity of 49.89kph (31mph) and impacts the vehicle structure. The CPU time for one nonlinear FE simulation using the RADIOSS software is approximately 20 hours on an SGI Origin 2000.

The optimal Latin hypercube sampling with a total of 33 runs was used to generate a sample of design points for construction of the stepwise regression response surface.

The explicit response used in the RBDO is summarized in Ref. 13. In this example, the explicit approximations of responses are regarded as exact responses of vehicle side impact to demonstrate the proposed adaptive probability analysis. A total of 11 random parameters are employed, as listed in Table 1, and the response of an abdomen load is used for adaptive probability analysis.
This study presents a comparison between LS and MLS methods for MPP locus approximation. As the probability levels are increased, a posteriori errors, $\varepsilon_1$ and $\varepsilon_2$ in Eq. (19), are measured to observe the rate of convergence in approximating MPP locus. From Figs. 4 and 5, it can be seen that the MLS method shows a faster rate of convergence than the LS method in MPP locus approximation. The reason is that the MLS method is capable of reproducing both local and global behaviors in MPP locus better than the LS method. Thus, the MLS method requires smaller number of probability levels compared to the LS method for probability analysis.

Example 2: Mathematical Example for Probability Analysis

A performance function for output probability analysis is defined as

$$G(X) = 0.3X_1^2X_2 - X_2 + 0.8X_1 + 1.0 \quad (20)$$

where two random parameters are defined as $X_1 \sim N(1.3,0.55)$ and $X_2 \sim N(1.0,0.55)$. As shown in Fig. 6, the performance function and the MPP locus are highly nonlinear. Moreover, the performance function is non-monotonic over a probability integration domain, which could result in large error in reliability analysis.

As shown in Fig. 7 and Table 2, the proposed adaptive probability analysis provides very accurate
probability function with only 57 analyses. Even though highly nonlinear and non-monotonic response is used for output probability analysis, the first- and second-order moment of response are fairly close to the numerically exact solution from Monte Carlo Simulation with 10,000 samples.

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Example 3: Side Impact Crashworthiness for Probability Analysis

The same crashworthiness example is again used in the proposed probability analysis of vehicle side impact. In this example, the response of the abdomen load is used for adaptive probability analysis.

As observed in Fig. 8 and Table 3, the proposed adaptive probability analysis provides very accurate probability distribution function of the abdomen load with only 33 analyses, which is so close to the result from Monte Carlo simulation (MCS) with one million samples. Note that the MCS with even 10,000-sample size does not provide the output probability result as accurate as the adaptive probability analysis method. Even though a large number of random parameters is used for probability analysis, the number of analyses required is small for the proposed adaptive probability analysis method. From Example 2 and Example 3, it can be seen that the number of required analyses in the proposed adaptive probability analysis method does not depend on the number of random parameters, but depend on the degree of nonlinearity of the performance function.

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**CONCLUSIONS**

This paper presents a new method for the output probability analysis, which aids a decision-making process by effectively identifying the effect of input uncertainties. It has been shown that numerical difficulties in existing probability analysis methods, such as efficiency and accuracy, can be resolved by employing the proposed adaptive probability analysis method. The comparison study between LS and MLS methods demonstrate that the MLS method converges faster in approximating the MPP locus, compared to the LS method. Two of numerical examples are used to show that the proposed adaptive probability analysis is very efficient and accurate. Also, it is found that numerical efficiency in adaptive probability analysis does not depend on the number of random parameters but the degree of nonlinearity of the performance function.
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