Shape Design Optimization of Thermoelastic Structures for Durability

Shape design sensitivity analysis (DSA) and optimization methods for the fatigue life of thermoelastic structural components are presented in this paper. A multifield fatigue life prediction method is used for crack initiation. The crack initiation life prediction is modeled using constant amplitude strain-life data and cyclic stress-strain curves. A hybrid DSA method is used for the fatigue life. The design sensitivities of the dynamic stress and the temperature field are obtained using analytical approaches. The design sensitivity is used to predict the dynamic stress and temperature of the perturbed design. Using predicted stress and temperature, the fatigue life of the perturbed structural design component is predicted. The predicted fatigue life is then used to obtain the design sensitivity of the fatigue life by utilizing the finite difference method. The proposed DSA method is applied to design optimization of an automotive exhaust manifold of an automotive vehicle, considering crack initiation lives as design constraints.

1. Introduction

The thermal stress [1-3] and structural durability [4, 5] are some of the most important concerns in designing powertrain components. High thermal stress combined with a variable mechanical load may initiate a fatigue crack and subsequently cause failure of components. In structural design, static stress concentration factors, instead of the fatigue life, are often used as criteria for durability. In this case, the worst case scenario is commonly employed to design for durability with stress concentration factors criteria, where stresses are obtained by applying a set of critical (peak) loading. This may yield wrong design criteria to determine optimum design since the high stress area identified at peak loads may not be the critical area where the crack initiates.

Objectives of this paper are to (1) present an efficient and accurate design sensitivity analysis (DSA) method for the fatigue life of thermoelastic structural components and (2) use design sensitivities to support design optimization considering the structural fatigue life as the design criterion. Methods developed in this paper assume steady-state temperature fields with variable mechanical loading.

To generate a representative mechanical load history, multibody dynamic analysis is performed for the mechanical system under a typical service cycle. In addition, steady state thermal analysis is performed to obtain temperature distribution in the structural component. Quasi-static structural finite element analyses are then performed to obtain stress influence coefficients of the structural component of the mechanical system corresponding to each external loading and thermal field. Stress influence coefficients are combined with mechanical loading histories obtained from multibody dynamic analysis to obtain the dynamic stress history. The stress history is then employed to predict the fatigue life of the component using the von Mises equivalent strain method [6]. A set of critical points is then identified and is used in defining the constraints for optimization problem.

In this paper, the continuum DSA method [7-10] is employed to compute design sensitivity coefficients of the thermal field, which are then used to compute design sensitivities of stress influence coefficients. These are used further to obtain the design sensitivity of the dynamic stress [11]. The proposed method of durability DSA utilizes the dynamic stress sensitivity and the temperature sensitivity to predict dynamic stress histories and temperature of the perturbed design component, it then computes the fatigue life of the perturbed design component at the critical points using the predicted dynamic stress and the predicted temperature field, and then uses the finite difference between the new life and the original life at the same critical points to approximate the design sensitivity of the structural component fatigue life. In this approach, it is assumed that the mechanical loading history is independent of design. This assumption is reasonable for structural components of mechanical systems since local design changes are quite effective means of increasing the fatigue life.

2. Analysis of Thermoelastic Systems and Life Prediction

In this section, variational forms of the steady state heat conduction equation and the elasticity equilibrium equation with thermal load are presented. Using the linearity, the stress time history is obtained by superposing stress influence coefficients and mechanical loading histories. The fatigue life is computed by the cumulative damage method. The computational flowchart is shown in Fig. 2.1.

Thermal Analysis. Consider a three-dimensional thermoelastic, isotropic and homogeneous solid, as shown in Fig. 2.2. The steady state heat conduction equation and boundary conditions are given as

\[
-k \frac{\partial \theta}{\partial r} = q \quad \text{in } \Omega \\
\theta = \theta_0 \quad \text{on } \Gamma_0^\theta \\
k \theta n^t = q \quad \text{on } \Gamma_1^\theta \\
k \theta n^t + h(\theta - \theta_s) = 0 \quad \text{on } \Gamma_2^\theta \tag{2.1}
\]

where \(\theta = T - T_0\), \(T\) is the absolute temperature, \(T_0\) is the reference temperature of the stress-free state of the solid body, \(\theta_s\) is the prescribed temperature, \(\theta_s\) is the ambient temperature,

---

$n^i$ is the $i$th component of the unit normal vector on the boundary, $k$ is the heat conductivity of the body, $h$ is the convective heat transfer coefficient, $q$ is the heat flux vector, $g$ is the internal heat source, $\Gamma_0^\theta$ is the boundary where the temperature is prescribed, $\Gamma_1^\partial$ is the boundary where the heat flux is prescribed, and $\Gamma_2^\partial$ is the boundary where the heat convection is prescribed.

Multiplying both sides of the heat conduction equation with a virtual temperature field $\hat{\theta}$, integrating over the physical domain $\Omega$, and integrating by parts and using the boundary conditions, yield

$$\begin{align*}
\int_\Omega k \cdot \nabla \hat{\theta} \cdot d\Omega + \int_{\Gamma_1^\partial} h \partial_{\hat{\theta}} d\Gamma &= \int_{\Gamma_2^\partial} g \hat{\theta} d\Gamma \\
&+ \int_{\Omega} \hat{\theta} \cdot q d\Omega + \int_{\Gamma_2^\partial} h \theta \hat{\theta} d\Gamma, \quad \text{for all } \hat{\theta} \in \Theta
\end{align*}$$

where $\Theta$ is the space of kinematically admissible temperature

$$\Theta = \{ \theta \in H^1(\Omega); \theta = 0, x \in \Gamma_0^\theta \}$$

and $H^1$ is the Sobolev space of order 1. The bilinear form and the linear form for the temperatures can be defined as

$$A(\theta, \hat{\theta}) = \int_{\Omega} k \cdot \nabla \theta \cdot \nabla \hat{\theta} d\Omega + \int_{\Gamma_1^\partial} h \partial_{\theta} \hat{\theta} d\Gamma$$

$$L(\hat{\theta}) = \int_{\Omega} g \hat{\theta} d\Omega + \int_{\Gamma_2^\partial} q \hat{\theta} d\Gamma + \int_{\Gamma_2^\partial} h \theta \hat{\theta} d\Gamma$$

The variational form of the heat equation is then rewritten as

$$A(\theta, \hat{\theta}) = L(\hat{\theta}), \quad \text{for all } \hat{\theta} \in \Theta$$

**Elastic Analysis.** The equilibrium equation and boundary conditions for a general three-dimensional elasticity can be written as

$$\begin{align*}
-\sigma^{ij} &= f^i \quad \text{in } \Omega \\
c^{ij} &= e^{ij} \quad \text{on } \Gamma_0^\theta \\
t^i &= \sigma^{ij} n^j \quad \text{on } \Gamma_2^\partial
\end{align*}$$

where $f^i$ is the $i$th component of the body force, $c^i$ is the $i$th component of the traction on the boundary, $t^i$ is the $i$th component of the displacement, $\Gamma_0^\theta$ is the boundary where the displacement is prescribed, and $\Gamma_2^\partial$ is the boundary where the traction force is prescribed. In Eq. 2.6, the stress tensor is defined as

$$\sigma^{ij}(z, \theta) = D^{\text{ij}}(T)[e^{ij}(z) - \delta^{im}\alpha \theta]$$

$$\sigma^{ij}(z, \theta) = D^{\text{ij}}(T)[e^{ij}(z) - \delta^{im}\alpha \theta]$$

where $\theta = T - T_0$, $e^{ij}(z) = (c_{ij} + z_{ij})/2$ is the strain tensor, $\alpha$ is the coefficient of linear thermal expansion, $\delta^{im}$ is the Kronecker delta, and $\beta(T) = \alpha E(T)/(1 - 2\nu(T))$ is the thermal modulus. The elasticity tensor in Eq. 2.7 is defined as

$$D^{\text{ij}}(T) = \lambda(T) \delta^{ij} \delta^{mn} + \mu(T)(\delta^{im} \delta^{jn} + \delta^{in} \delta^{jm})$$

where $\lambda(T) = (\nu(T) E(T))/(1 + \nu(T))(1 - 2\nu(T))$ and $\mu(T) = (E(T)/2(1 + \nu(T)))$ are Lamé's constants, and $E(T)$ and $\nu(T)$ are the Young's modulus and Poisson's ratio, respectively. Note that the Young's modulus and Poisson's ratio depend on the absolute temperature $T$.

The weak form of the elasticity equation is

$$\int_\Omega \sigma^{ij}(z, \theta) e^{ij}(z) d\Omega = \int_\Omega f^i e^{ij} d\Omega + \int_{\Gamma_2^\partial} t^i e^{ij} d\Gamma$$

for all $z \in Z$ (2.9)

where $Z$ is the space of kinematically admissible virtual displacements

$$Z = \{ z \in [H^1(\Omega)]^2; \quad z^i = 0, x \in \Gamma_0^\theta \}$$

Using stress-strain relationship from Eq. 2.7, the variational form of Eq. 2.9 becomes

$$\int_\Omega D^{\text{ij}}(T) e^{ij}(z) e^{ij}(z) d\Omega = \int_\Omega (f^i e^{ij} + \beta(T) \theta t^i) d\Omega$$

$$+ \int_{\Gamma_2^\partial} t^i e^{ij} d\Gamma, \quad \text{for all } z \in Z$$

The thermal problem and elasticity problem are decoupled, so the elasticity problem is still linear, even though the Young's modulus and Poisson's ratio are assumed to be dependent on temperature. By defining energy bilinear form as

$$a(z, \zeta) = \int_\Omega D^{\text{ij}}(T) e^{ij}(z) e^{ij}(z) d\Omega$$

and load linear form as

$$l(z) = \int_\Omega (f^i e^{ij} + \beta(T) \theta t^i) d\Omega$$

Eq. 2.11 becomes

$$a(z, \zeta) = l(z), \quad \text{for all } z \in Z$$

**Linear Elastic Stress Superposition.** The formulas for stresses developed in the preceding section are valid only for static thermal load and static mechanical load. As shown in Ref. 6, the dynamic stresses can be computed using a quasi-static method in the case of transient dynamic loads, if the frequency of the applied loads is substantially below the natural frequency of the structural component. The key idea is to decompose the total transient dynamic load into a linear combination of quasi-static loads with time dependent coefficients. Superposition can then be applied, if the essential boundary conditions are homogeneous. In such a case, the solution space is the same as space $Z$ from Eq. (2.10).

When the thermal loads are involved, however, the superposition method must be applied with caution. If material prop-
Fig. 2.3 Crack initiation life prediction procedure

Fig. 3.2 Dependence of fatigue life on shape design parameters

\[ a(z, x) = l_{s}(x), \text{ for all } x \in Z \]  \hspace{1cm} (2.18)

with \( s = 1, 2, \ldots, N \). The bilinear forms in Eqs. 2.17 and 2.18 are the same as the bilinear form in Eq. 2.14. The solution \( z \) of Eq. 2.14 can be obtained by superposition as

\[ z = \varphi_{1}(t)z_{1} + \varphi_{2}(t)z_{2} + \ldots + \varphi_{N}(t)z_{N} + z_{0} \]  \hspace{1cm} (2.19)

From the linearity of the elasticity tensor and strain tensor, the total transient dynamic stress tensor can be obtained as

\[ \sigma^{ij}(x, \theta, t) = D^{ijm}(T)e^{mm}(z_{0}) - \beta(T)\delta^i\delta^j + \sum_{s=1}^{N} \varphi_{s}(t)D^{ijm}(T)e^{mm}(z_{s}) \]  \hspace{1cm} (2.20)

For the thermal load case, using Eq. 2.7, the stress is given by

\[ \sigma^{ij}(z_{0}, \theta) = \sigma^{ij}(z_{0}, \theta) + \sum_{s=1}^{N} \varphi_{s}(t)D^{ijm}(T)e^{mm}(z_{s}) \]  \hspace{1cm} (2.21)

so Eq. 2.19 can be written in a simplified form as

\[ \sigma^{ij}(x, \theta, t) = \sigma^{ij}(z_{0}, \theta) + \sum_{s=1}^{N} \varphi_{s}(t)D^{ijm}(T)e^{mm}(z_{s}) \]  \hspace{1cm} (2.22)

Note that if the total mechanical load can be decomposed into \( N \) loads, then the elasticity problem has to be solved \( (N + 1) \) times. The dynamic stress history in Eq. 2.22 is used to predict the fatigue life of the structural component.

**Fatigue Life Prediction.** The fatigue is a process of initiating and growing a crack which finally causes failure of a structural component. The mathematical models used to simulate the crack initiation and propagation processes are quite different. This paper will be concerned with crack initiation life predic-

---

**Fig. 3.1** Deformation process

**Fig. 3.3** Flowchart for DSA of fatigue life

Journal of Mechanical Design

SEPTEMBER 1998, Vol. 120 / 493
tion, although when the stress history is given, the crack growth can also be estimated [5]. The crack initiation life prediction is based on the multiaxial fatigue initiation life prediction method and is modeled using von Mises equivalent strain approach with Smith-Watson-Topper theory.

A number of life prediction methods, such as von Mises equivalent strain, ASME Boiler Code, and Tensile and Shear Critical Plane methods are commonly used and have been implemented in the Durability and Reliability Analysis Workspace (DRAW) [6, 13]. These methods attempt to correlate a known local elastic-plastic strain state with the crack initiation life. The initiation life prediction is given in terms of the number of irregular loading history repetitions which cause a crack length of 2 mm. The crack initiation life computational procedure is illustrated in Fig. 2.3.

3. Shape Design Sensitivity Analysis of Thermoelastic Systems for Durability

A method of computing the design sensitivity of the fatigue life performance measure of the thermoelastic system is presented in this section. The shape design sensitivity of the stress performance measure is obtained by differentiating the variational forms of the thermal equation and elasticity equation using the material derivative concept developed in Refs. 7 and 8. This continuum DSA method is extended to compute the design sensitivity of the dynamic stress history, which is used to predict changes in dynamic stress history due to design changes [11]. The fatigue life of the component is computed for the perturbed design using the predicted stress history and the predicted temperature field from DSA. The design sensitivity of the structural component fatigue life is then obtained using the finite difference method.

**Design Sensitivity Analysis of Thermoelastic Systems.**

Considering the structural domain as a continuous medium and the process of changing the shape of domain \( \Omega \) to \( \Omega_\tau \) in Fig. 3.1 as a dynamic process that deforms the continuum with \( \tau \) playing the role of time, a transformation mapping \( T \) that represents this process can be defined as [7, 8, 14]

\[
T : x \rightarrow x(\tau), \quad x \in \Omega
\]

with

\[
x_\tau = T(x, \tau) \\
\Omega_\tau = T(\Omega, \tau) \\
\Gamma_\tau = T(\Gamma, \tau)
\]

Suppose that a material point \( x \in \Omega \) in the initial domain at \( \tau = 0 \) moves to a new location \( x_\tau \in \Omega_\tau \) in the perturbed domain. A design velocity field \( V \) can then be defined as

\[
V(x_\tau, \tau) = \frac{dx_\tau}{d\tau} = \frac{dT(x, \tau)}{d\tau} = \frac{\partial T(x, \tau)}{\partial \tau}
\]

since the initial point \( x \) does not depend on \( \tau \). In the neighborhood of initial time \( \tau = 0 \), assuming a regularity hypothesis and ignoring higher-order terms, \( T \) can be approximated by

\[
T(x, \tau) = T(x, 0) + \tau \frac{\partial T(x, 0)}{\partial \tau} + O(\tau^2)
\]

\[
\approx x + \tau V(x, 0)
\]

where \( x = T(x, 0) \) and \( V(x) = V(x, 0) \). The pointwise material derivative at \( x \in \Omega \) is defined as

\[
z = \lim_{\tau \to 0} \frac{z(x + \tau V) - z}{\tau} = \lim_{\tau \to 0} \frac{z(x + \tau V) - z(x + \tau V)}{\tau}
\]

\[
+ \lim_{\tau \to 0} \frac{z(x + \tau V) - z}{\tau} = z' + z_\tau V^2
\]

Table 4.1 Elastic material properties

<table>
<thead>
<tr>
<th>Temperature (Celsius)</th>
<th>Young's modulus (N/mm²)</th>
<th>Poisson's ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>150</td>
<td>165000</td>
<td>0.300</td>
</tr>
<tr>
<td>200</td>
<td>153000</td>
<td>0.290</td>
</tr>
<tr>
<td>250</td>
<td>150000</td>
<td>0.290</td>
</tr>
<tr>
<td>300</td>
<td>148000</td>
<td>0.290</td>
</tr>
<tr>
<td>350</td>
<td>141000</td>
<td>0.290</td>
</tr>
<tr>
<td>400</td>
<td>137000</td>
<td>0.290</td>
</tr>
<tr>
<td>450</td>
<td>133000</td>
<td>0.290</td>
</tr>
<tr>
<td>500</td>
<td>130000</td>
<td>0.290</td>
</tr>
<tr>
<td>550</td>
<td>127000</td>
<td>0.290</td>
</tr>
<tr>
<td>600</td>
<td>120000</td>
<td>0.290</td>
</tr>
<tr>
<td>650</td>
<td>117000</td>
<td>0.290</td>
</tr>
<tr>
<td>700</td>
<td>112000</td>
<td>0.290</td>
</tr>
<tr>
<td>750</td>
<td>109000</td>
<td>0.290</td>
</tr>
<tr>
<td>816</td>
<td>105000</td>
<td>0.290</td>
</tr>
<tr>
<td>1010</td>
<td>250000</td>
<td>0.290</td>
</tr>
</tbody>
</table>

Fig. 4.1 Thermoelastic model

494 / Vol. 120, SEPTEMBER 1998

Transactions of the ASME
where $x'$ is the partial derivative of $x$ with respect to design variable $\tau$. Assuming reasonable smoothness, the partial derivative with respect to $\tau$ and the partial derivative with respect to $x'$ commute, i.e.,

$$\frac{\partial}{\partial \tau} \left( \frac{\partial z}{\partial x'} \right) = \frac{\partial}{\partial x'} \left( \frac{\partial z}{\partial \tau} \right) = \frac{\partial x'}{\partial x}$$

(3.6)

The domain functional and its material derivative are
\[
\psi_1 = \int_{\Omega} F d\Omega \\
\psi_2 = \int_{\Gamma} G d\Gamma
\]

and the boundary functional and its material derivative are

\[
\psi_2 = \int_{\Gamma} [G' + (G_r n^i + HG)V^n] d\Gamma
\] (3.7)

where \( H \) is the curvature of the design boundary.

Taking the material form of both sides of Eq. 2.5, assuming \((\theta h)_{\theta} \theta = 0, g' = q' = 0, \) and choosing the path \( \theta = 0, \) the following equation can be obtained

\[
A(\theta, \theta) = L(\theta) - A(\theta, \theta), \quad \text{for all } \theta \in \Theta \quad (3.9)
\]

where the bilinear form \( A \) and the linear form \( L \) are defined by Eq. 2.4,

\[
A(\theta, \theta) = \int_{\Omega} \left[ (k(\theta, V^i) \theta_{\theta} + k\theta_{\theta} V^i - k\theta_{\theta} V^i) \right] d\Omega
\]

\[
- \int_{\Gamma} \left[ (h\theta_{\theta} V^k + h\theta_{\theta} V^k) + [(h\theta_{\theta}) + H h\theta_{\theta}] V^n] d\Gamma \quad (3.10)
\]

and

\[
L(\theta) = \int_{\Omega} \left[ g_s \theta_{\theta} V^k + g_s \theta_{\theta} V^k \right] d\Omega + \int_{\Gamma} \left[ q(-\theta V^k) + [(q_{\theta}) + H q_{\theta}] V^n d\Gamma \right.
\]

\[
+ [(q_{\theta}) + H q_{\theta}] V^n d\Gamma \right] + \int_{\Gamma} \left[ h\theta_{\theta} (-\theta V^k) + [(h\theta_{\theta}) + H h\theta_{\theta}] V^n d\Gamma \right]
\] (3.11)

Comparing Eq. 2.5 and Eq. 3.9, it is noted that the bilinear forms on the left-hand side are identical with different arguments \( \theta \) and \( \theta \) in Eqs. 2.5 and 3.9, respectively) and that the right-hand sides are different. In FEA, they can be considered as two different loading cases with the same stiffness matrix. Equation 2.5 is solved first to obtain \( \theta \) and the decomposed stiffness matrix is reused to obtain \( \theta \) using Eq. 3.9.

For the elasticity part, taking the material derivative of both sides of Eq. 2.14, assuming \((f')' = (f')' = 0, \) and choosing the path \( \theta = 0, \) the following equation can be derived

\[
\frac{\partial}{\partial \Omega} \left( \frac{\partial L(\theta)}{\partial \theta} \right) = \chi(z, \theta, \theta) + \frac{\partial L(\theta)}{\partial \theta} - \frac{\partial L(\theta)}{\partial \theta} - \frac{\partial \chi(z, \theta, \theta)}{\partial \theta}
\]

where

\[
\chi(z, \theta, \theta) = \int_{\Omega} \left[ -\frac{\partial}{\partial \theta} (D(\theta)) \frac{\partial e}{\partial \theta} \right] d\Omega
\]

\[
+ \int_{\Omega} \left[ \frac{\partial}{\partial \theta} \left( T(\theta) \right) \right] \frac{\partial e}{\partial \theta} d\Omega
\] (3.13)

\[
i'\chi(z) = \left[ f' + f' V^n \right] d\Omega + \int_{\Gamma} \left[ t'(-\theta V^k) \right]
\]

\[
+ \left[ (t'V^k) n^k + H t' n^k \right] V^n d\Gamma \quad (3.14)
\]

Similar to its thermal counterpart, the elasticity equation Eq. 2.11 is first solved for displacement \( z \) and subsequently the

**Table 4.3 Verification of design sensitivity coefficients of life**

<table>
<thead>
<tr>
<th>Node Id</th>
<th>L(b+\theta b)</th>
<th>L(b)</th>
<th>L(b+\theta b)-L(b)</th>
<th>dL/\theta b</th>
<th>dL/L %</th>
<th>Change %</th>
</tr>
</thead>
<tbody>
<tr>
<td>3331</td>
<td>.7721602E+07</td>
<td>.7466350E+07</td>
<td>.25535E+06</td>
<td>.25536E+08</td>
<td>99.9</td>
<td>3.42</td>
</tr>
<tr>
<td>3549</td>
<td>.9520163E+07</td>
<td>.8593949E+07</td>
<td>.56621E+06</td>
<td>.57732E+08</td>
<td>101.9</td>
<td>6.32</td>
</tr>
<tr>
<td>3546</td>
<td>.417754E+08</td>
<td>.364464E+08</td>
<td>.53291E+08</td>
<td>.53291E+08</td>
<td>100.0</td>
<td>3.90</td>
</tr>
<tr>
<td>3266</td>
<td>.159494E+08</td>
<td>.145534E+08</td>
<td>.4409E+06</td>
<td>.46629E+08</td>
<td>104.9</td>
<td>3.13</td>
</tr>
<tr>
<td>3529</td>
<td>.1713074E+08</td>
<td>.1619933E+08</td>
<td>.10214E+07</td>
<td>.10634E+09</td>
<td>102.1</td>
<td>6.34</td>
</tr>
<tr>
<td>3333</td>
<td>.309752E+08</td>
<td>.3039790E+08</td>
<td>.57732E+06</td>
<td>.57732E+08</td>
<td>100.0</td>
<td>1.89</td>
</tr>
<tr>
<td>3549</td>
<td>.3529288E+08</td>
<td>.3328448E+08</td>
<td>.19984E+07</td>
<td>.20428E+09</td>
<td>102.2</td>
<td>1.60</td>
</tr>
<tr>
<td>3282</td>
<td>.3732857E+08</td>
<td>.3777298E+08</td>
<td>.4409E+05</td>
<td>.4409E+07</td>
<td>100.0</td>
<td>-.13</td>
</tr>
<tr>
<td>3552</td>
<td>.5146564E+08</td>
<td>.4960476E+08</td>
<td>.1852E+09</td>
<td>.19540E+09</td>
<td>104.7</td>
<td>3.76</td>
</tr>
<tr>
<td>3480</td>
<td>.1852740E+09</td>
<td>.1827871E+09</td>
<td>.24869E+07</td>
<td>.24869E+09</td>
<td>100.0</td>
<td>1.36</td>
</tr>
</tbody>
</table>

**Fig. 4.4 Optimization flowchart**
Stress history design sensitivity can then be evaluated by taking the material derivative of Eq. 2.22 as

$$\frac{d}{d\tau} (\sigma^u(z, \theta, \tau)) = \frac{d}{d\tau} (\sigma^u(z_0, \theta))$$

$$+ \sum_{i=1}^{N} \phi_i(\tau) \frac{d}{d\tau} (D^{imn}(T)\varepsilon^{mn}(z_i))$$  (3.17)

**Design Sensitivity Analysis of Fatigue Life.** The finite difference method is used to compute the design sensitivity of the component fatigue life. An analytical approach for fatigue life DSA is difficult since the fatigue life is obtained after performing peak-valley editing and rainflow counting procedures. Figure 3.2 shows the dependency of the fatigue life on design variables. Figure 3.3 shows the computational flowchart of the fatigue life DSA.

Once the sensitivity of the transient stress history is obtained using the DSA method described above, increments of stress history can be obtained by

$$\delta\sigma^u(z, \theta, \tau) = \frac{d}{d\tau} (\sigma^u(z, \theta, \tau)) \delta\tau$$  (3.18)

where $\delta\tau$ is step size. The step size $\delta\tau$ must be small for linear approximation of the fatigue life. On the other hand, in numerical calculation, $\delta\tau$ cannot be too small since it may introduce numerical noise. The transient stress history of the perturbed design can be approximated by

$$\sigma(\tau + \delta\tau) = \sigma(\tau) + \delta\sigma(\tau)$$  (3.19)

where $\sigma$ is the stress tensor. The perturbed temperature field is also needed for life computation of the component. This perturbed temperature field is easier to compute, because the temperature field does not depend on time $t$, it depends only on the design parameter $\tau$ as

$$\theta(\tau + \delta\tau) = \theta(\tau) + \frac{d\theta}{d\tau} \delta\tau$$  (3.20)

The new transient stress history is then used to calculate the
fatigue life of the component with a perturbed design, \( L(\tau + \delta\tau) \), using methods discussed in Section 2. The design sensitivity coefficient of the component fatigue life with respect to the design parameter \( \tau \) can be obtained from

\[
\frac{\partial L}{\partial \tau} \approx \frac{L(\tau + \delta\tau) - L(\tau)}{\delta\tau}
\]  

(3.21)

Note that Eqs. 3.18 to 3.21 must be evaluated for each design parameter.

4. Numerical Example—An Engine Exhaust Manifold

An engine exhaust manifold of a ground vehicle is employed to demonstrate fatigue life analysis and DSA methods proposed in this research. This component of the engine is subjected to a large thermal load due to the high temperature of the exhaust gas, in addition to variable dynamic mechanical load. The combination of these loads affects the fatigue life of the manifold significantly.

Thermoelastic Model. Reports from the automotive industry show that the exhaust manifold being investigated cracks during testing. To simulate the test process, dynamic analysis and finite element analysis (FEA) are employed. The analysis procedures are decoupled into thermal analysis, transient stress analysis, and life prediction.

The manifold is made of cast iron, with an average thermal conductivity of \( k = 0.0026 \text{ W/mm/K} \) [15]. The average temperature of the exhaust gas is 1100°C. The temperature of the cooling outer gas is 23°C. Figure 4.1 shows the thermal model with the thermal boundary conditions. The thermal load is assumed to be convection only, and so the entire surface boundary is treated as \( \Gamma^i \) (see Eq. 2.1). The experimental data were not available for this manifold, so the convection film coefficients are computed using approximate formulas obtained from Ref. 16. The film coefficient on the inner surface is \( h_{int} = 1.148 \times 10^{-5} \text{ W/mm}^2/\text{°C} \) and on the outer surface is \( h_{ext} = 2.148 \times 10^{-6} \text{ W/mm}^2/\text{°C} \). The output from the thermal analysis is the temperature field, which is computed using FEA.

For the elastic model, the dynamic load is simulated in an interval of 1.453 seconds using a sinusoidal function. The load is applied at the junction between the exhaust manifold and the muffler, as shown in Fig. 4.1. Elastic material properties are dependent on the temperature (Table 4.1) and linear interpolation is used. The temperature field is applied as a thermal load. The peak dynamic load is 720 N. The expansion coefficient is \( \alpha = 1.25 \times 10^{-5} /\text{°C} \) and the stress-free temperature (reference temperature) is \( T_0 = 23 \text{°C} \).

The ANSYS finite element model [17] contains 8,188 nodes and 4,782 elements. Among these, 3,685 are solid 8-node elements and 1,097 are shell 4-node elements. Some of the elements are degenerated as wedges (solid) and triangles (shell). The same finite element model is used for thermal analysis and elasticity analysis. For the thermal problem, the convection loads are applied on the inner and outer surface of the manifold and on interface between engine cylinders and the exhaust manifold. For the elasticity problem, the displacements in z-direction at the nodes on the interface with the engine are imposed to be

![Fatigue life contour in the critical area for optimum design](image)

Table 4.4 Cost and first 9 constraints at initial and optimum design

<table>
<thead>
<tr>
<th>Function</th>
<th>Description</th>
<th>Initial design</th>
<th>Optimum design</th>
<th>% changes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost</td>
<td>Volume</td>
<td>524409 mm³</td>
<td>513278 mm³</td>
<td>-2.122</td>
</tr>
<tr>
<td>Constraint 1</td>
<td>Life, node 3331</td>
<td>7.466250e+06</td>
<td>9.920953e+07</td>
<td>228.7</td>
</tr>
<tr>
<td>Constraint 2</td>
<td>Life, node 3549</td>
<td>8.953949e+06</td>
<td>8.077982e+07</td>
<td>802.1</td>
</tr>
<tr>
<td>Constraint 3</td>
<td>Life, node 3546</td>
<td>1.364464e+07</td>
<td>7.118750e+07</td>
<td>431.6</td>
</tr>
<tr>
<td>Constraint 4</td>
<td>Life, node 3266</td>
<td>1.415534e+07</td>
<td>1.040056e+08</td>
<td>634.7</td>
</tr>
<tr>
<td>Constraint 5</td>
<td>Life, node 3329</td>
<td>1.610933e+07</td>
<td>4.622968e+07</td>
<td>186.9</td>
</tr>
<tr>
<td>Constraint 6</td>
<td>Life, node 3333</td>
<td>3.039790e+07</td>
<td>3.328448e+07</td>
<td>1179.8</td>
</tr>
<tr>
<td>Constraint 7</td>
<td>Life, node 3548</td>
<td>3.328448e+07</td>
<td>4.259703e+08</td>
<td>1750.0</td>
</tr>
<tr>
<td>Constraint 8</td>
<td>Life, node 3282</td>
<td>3.377298e+07</td>
<td>4.951594e+07</td>
<td>46.61</td>
</tr>
<tr>
<td>Constraint 9</td>
<td>Life, node 3552</td>
<td>4.960476e+07</td>
<td>6.966871e+08</td>
<td>304.5</td>
</tr>
</tbody>
</table>

Note that Eqs. 3.18 to 3.21 must be evaluated for each design parameter.
zero. To avoid rigid body movement, two nodes have additional constraints. The dynamic load is applied at the nodes near the bolts that connect the muffler and the exhaust manifold. Forces from the attachment bolts to the engine are neglected.

The temperature field obtained using the ANSYS thermal analysis [18] is applied as the thermal load for the elasticity analysis. Thermal analysis contains a single load case, corresponding to the highest temperature of the exhaust gas, while elasticity analysis contains two load cases, one corresponding to the maximum mechanical load applied in the negative x-direction, and the second corresponding to the maximum load applied in the positive x-direction. The thermal load is applied in both load cases, because the stiffness matrix corresponding to bilinear form contains terms that depend on the temperature. The total dynamic stress tensor is obtained from

\[
\sigma^{(q)}(z, \theta, t) = \varphi_1(t)\sigma^{(a)}(z, \theta) + \varphi_2(t)\sigma^{(b)}(z, \theta) \tag{4.1}
\]

where \(z\) and \(z_t\) are the displacement fields for two load cases considered.

Fatigue life is calculated using a von Mises equivalent strain method that is implemented in DRAW [6, 13]. The material properties for the corresponding uniaxial model are fatigue strength coefficient 807 N/mm^2, fatigue strength exponent -0.08, fatigue ductility coefficient 0.29, fatigue ductility exponent -0.60. For the local elastic-plastic effect, Ramberg-Osgood equation is involved [5], with cyclic strength coefficient 800 N/mm^2 and cyclic strain hardening exponent 0.12.

It is interesting to note that the lowest fatigue life does not appear in the region where the highest stress corresponding to the peak load occurs. Similar observation has been reported also in Ref. 10 for components with mechanical loads only. This behavior is due to the fact that the fatigue life depends on the variation of stresses in time as well as their absolute magnitude at the peak load. A fatigue life contour for the initial design is shown in Fig. 4.2 (for convenience, decimal logarithm of fatigue life is displayed). Table 4.2 contains the life at the most critical nodes. The results show that the lowest life is 7.46625 × 10^6 blocks for the initial design. For a block with 1,453 seconds, this corresponds to one year, when the average use is eight hours per day, seven days per week.

Most of the nodes with a low fatigue life are concentrated in the area between the fourth cylinder outlet and the junction with the muffler. This area is parameterized in an attempt to increase the life of the component after proper design changes. The geometric model is created using P3/PATRAN [19]. Eight shape design parameters are selected for this problem as shown in Fig. 4.3. Design parameters 1–4 characterize the inner and outer cross sections of two cylindrical cross sections AA and BB of the manifold; design parameters 5–8 characterize the inner and outer cross section of the junction area between the fourth runner and main exhaust pipe, i.e., CC in Fig. 4.3. Note that bicubic patches are employed to represent boundary geometry of the manifold segment shown in Fig. 4.3. Eight corresponding design velocity fields are computed using the isoparametric mapping method [9, 10] implemented in the Design Sensitivity Analysis and Optimization (DSO) Tool [20].

Design Sensitivity Analysis and Result Verifications. Design sensitivity coefficients of stress components with respect to eight design parameters are computed using the direct differentiation method [7, 13]. The accuracy of the fatigue life sensitivity is verified at nodes with low fatigue life, using the overall finite difference. In Table 4.3, \(L(\tau)\) and \(L(\tau + \delta\tau)\) are fatigue lives at the initial and perturbed designs, considering a 0.01 mm perturbation of design parameter \(dp5: \delta L\) is the perturbation obtained from the overall finite difference method; \(L'\) is the predicted perturbation using design sensitivity, \(L' = \delta L/dp5 \times \delta p5\), and \(dL/dL'\) is accuracy of the design sensitivity. A value close to 100% indicates accurate design sensitivity.

Design Optimization. The shape of the exhaust manifold is optimized using the shape design parameters defined. The objective of the design problem is to minimize the volume while increasing the durability from one year to six years. At the initial design, the structural volume is 5.24409 × 10^5 mm^3. Fatigue lives at the most critical 100 nodes are defined as constraints with the lower bound of 4.628 × 10^6 blocks. This lower bound is equivalent to 6 years of service life, assuming that the engine is operated eight hours per day, seven days per week. At the initial design, the lowest fatigue life is found at node 3331 with 7.46625 × 10^6 blocks to failure, which corresponds to 1.03 years of service life. Design optimization is performed using ANSYS [17, 18] as the FEA code, Design Optimization Tool (DOT) [21] for optimization, DRAW [13] for fatigue life computation. P3/PATRAN [19] is used as pre- and post-processor. The modified feasible direction method of DOT is employed for optimization. Figure 4.4 shows the computational flowchart.

Initial design was infeasible, since eight constraints were violated. A feasible design was obtained after six iterations. The cost function was further reduced, while keeping the design feasible after the sixth iteration. Fourteen successful iterations, with 134 finite element analysis and 14 design sensitivity computations were completed. After 14 iterations, ANSYS FEA could not be performed further without remeshing, which was not pursued. The optimization histories of the cost, design parameters and the first 9 normalized constraints are shown in Figs. 4.5 and 4.6. Table 4.4 shows the cost and fatigue lives at the initial and optimum designs, and percentage changes.

It can be noted that the fatigue life is improved significantly, with little change in volume. During the optimization process, the fatigue life was computed at 100 nodes, at which fatigue lives were low at the initial design. At the optimum design, fatigue lives were computed again for all the nodes of the model to verify results. The lowest fatigue life at the final design is found at node 3329 with 4.62968 × 10^6 blocks to failure, which corresponds to 6.4 years of service life. The life contour plot of the optimum design is shown in Fig. 4.7 (decimal logarithm of fatigue life is displayed).

5. Conclusions

An effective DSA and optimization method has been developed for durability-based design of thermoelastic structures. The current implementation assumes that the temperature field is not changed in time, while the mechanical load, which is variable in time, can be obtained from dynamic analysis. The dynamic stress history is obtained by the superposition of stresses computed from quasi-static linear analyses. Taking into account the fact that usually the temperature field varies much slower in time than the mechanical load, the assumption of steady-state temperature field may not be limiting for industrial applications. An engine exhaust manifold example was presented to demonstrate feasibility of the proposed method.

The following are future plans for study on DSA and optimization for durability of thermoelastic structures: (1) The proposed DSA method for steady-state thermal analysis is limited to linear problems, where the convective film coefficients and thermal conductivity are independent of temperature. It is desirable to extend the design sensitivity analysis to nonlinear thermoelastic models that can include also radiative loads, or even to extend to transient nonlinear thermal analysis; (2) All the bodies of the dynamic model are assumed to be rigid. If some of them are flexible, the DSA method should be extended; and (3) The proposed method for stress and durability is limited to linear elastic structures, even though the fatigue life is computed using the assumption of localized elastic-plastic behavior. Further study should take into account these nonlinear models.
References


