Solution Part B

(Total: 2 Parts)

57:021 Principles of Design I (Fall 2003)

Final Exam: 16 December 2003; Time: 2:15 to 4:15 PM

Name:_____________________

Section No. (*check one*)	☐ Arora 10:30 AM ☐ Choi 2:30 PM

Instructions:

1. **TWO** handwritten formula sheet is allowed.
2. If you need more space to answer a question, use **back of that question sheet only.**
3. The exam is divided into two parts. Submit each part separately.
4. Write your **name** and **section #** on each part.
5. S**how** all calculations and **justify** your answers. State all **assumptions**.

1.________/10

2.________/12

3. ________/10

4. ________/10

5. ________/08

Total:___________/50

Solution Part B

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1. (10 points) An engineering design problem is formulated as follows:

Maximize $z = -2x_1 + 3x_2$

subject to

$$6 - 3x_1 + 2x_2 \ge 0$$
$$-x_1 + x_2 - 4 \ge 0$$
$$x_1 \text{ is free in sign; } x_2 \ge 0$$

(i) Transcribe the problem to the standard LP format.

Minimize $f = 2\left(x_1^+ - x_1^-\right) - 3x_2$; $\qquad x_1 = x_1^+ - x_1^-$

$3\left(x_1^+ - x_1^-\right) - 2x_2 + x_3 = 6;$ $\qquad x_3 = slack\ variable$

$-\left(x_1^+ - x_1^-\right) + x_2 - x_4 + x_5 = 4;$ $\qquad x_4 = surplus\ variable$

$x_1^+, x_1^-, x_2 \ge 0;$ $\qquad x_5 = artificia;\ variable$

(ii) Set up the initial tableau for the problem so that the Simplex method can be used to solve the problem. *DO NOT SOLVE THE PROBLEM.*

Artificial cost, $w = x_5 = 4 + x_1^+ - x_1^- - x_2 + x_4$

	x_1^+	x_1^-	x_2	x_3	x_4	x_5	**b**
x_3	3	-3	-2	1	0	0	6
x_5	-1	1	1	0	-1	1	4
Cost	2	-2	-3	0	0	0	f - 0
Artificial cost	1	-1	1	0	1	0	w - 4

(iii) What are the initial values for the basic and nonbasic variables?

Basic: $x_3 = 6$, $x_5 = 4$; all others are nonbasic

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2. (12 points) A linear design optimization problem is to be solved using the Simplex method. The initial tableau for the problem is given below, where x_3 is a slack variable, x_4 is a surplus variable, and x_5 is an artificial variable.

	x_1	x_2	x_3	x_4	x_5	**b**
x3	-2	1	1	0	0	4
x5	1	1	0	-1	1	2
Cost	-4	-2	0	0	0	f - 0
Artificial cost	-1	-1	0	1	0	w - 2
x3	0	3	1	-2	2	8
x1	1	1	0	-1	1	2
Cost	0	2	0	-4	4	f+8
Art.	0	0	0	0	1	w-0

ANSWER THE FOLLOWING QUESTIONS.

(i) Identify basic and nonbasic variables for the initial tableau.

Basic: x3, x5; Nonbasic: x1, x2, x4

(ii) Complete (ONLY) one iteration of the Simplex method to obtain the next tableau.

(iii) Does <u>your tableau</u> give an optimum solution for the linear design problem? YES/NO

(iv) Does <u>your tableau</u> indicate end of Phase I of the Simplex method? YES/NO

(v) Does <u>your tableau</u> indicate the problem to be infeasible? YES/NO

(vi) Does <u>your tableau</u> indicate the problem to be unbounded? YES/NO

Solution Part B

(Total: 2 Parts)

2. (12 points) A linear design optimization problem is to be solved using the Simplex method. The initial tableau for the problem is given below, where x_3 is a slack variable, x_4 is a surplus variable, and x_5 is an artificial variable.

	x_1	x_2	x_3	x_4	x_5	**b**
x3	-2	1	1	0	0	4
x5	1	1	0	-1	1	2
Cost	-4	-2	0	0	0	f - 0
Artificial cost	-1	-1	0	1	0	w - 2
x3	-3	0	1	1	-1	2
x2	1	1	0	-1	1	2
Cost	-2	0	0	-2	2	f+2
Art	0	0	0	0	1	w-0

ANSWER THE FOLLOWING QUESTIONS.

(ii) Identify basic and nonbasic variables for the initial tableau.

Basic: x3, x5; Nonbasic: x1, x2, x4

(ii) Complete (ONLY) one iteration of the Simplex method to obtain the next tableau.

(vii) Does your tableau give an optimum solution for the linear design problem? YES/NO

(viii) Does your tableau indicate end of Phase I of the Simplex method? YES/NO

(ix) Does your tableau indicate the problem to be infeasible? YES/NO

(x) Does your tableau indicate the problem to be unbounded? YES/NO

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3. (10 points) A linear engineering design problem is formulated as follows:

Minimize $f = 10x_1 + 6x_2 + 2x_3 - 6x_4$

subject to

$$-x_1 + x_2 + x_3 - x_4 \geq 1$$

$$3x_1 + x_2 - x_3 - 3x_4 \geq 2$$

$$x_1, x_2, x_3, x_4 \geq 0$$

The final Simplex tableau for the problem is given as follows:

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	**b**
x_2	0	1	0.5	-1.5	-0.75	-0.25	0.75	0.25	1.25
x_1	1	0	-0.5	-0.5	0.25	-0.25	-0.25	0.25	0.25
Cost	0	0	4	8	2	4	-2	-4	f - 10
Artificial	0	0	0	0	0	0	1	1	w - 0

x_5 and x_6 are surplus variables for constraint #1 and 2, respectively. x_7 and x_8 are artificial variables for constraint #1 and 2, respectively.

ANSWER THE FOLLOWING QUESTIONS.

i	x_1 is a nonbasic variable	YES/**NO**
ii	Value of x_2 at the optimum point is 0.25	YES/**NO**
iii	Value of x_6 is zero at the optimum point	**YES**/NO
iv	Value of the cost function at the optimum is 10	**YES**/NO
v	Constraint #1 is inactive at the optimum point	YES/**NO**
vi	Constraint #2 is active at the optimum point	**YES**/NO
vii	Lagrange multiplier for constraint #1 is 2	YES/**NO**
viii	Lagrange multiplier for constraint #2 is -4	**YES**/NO
ix	The problem has multiple optimum points	YES/**NO**
x	Cost coefficient of x_3 can be increased to 4 without affecting the optimum cost function	**YES**/NO

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4. (10 points) An engineering design problem is formulated in terms of the design variables x and y as follows:

Minimize $f = x\left(\frac{1}{2}x - y^2\right)$

Subject to

$$h_1 : \ y(x + 4y) = 10$$

$$g_1 : \ -2x^2 + \tfrac{1}{3}y \le 0$$

$$g_2 : \ -y - 4 \le 0$$

(i) Evaluate cost and constraint functions at the point (2,3).

$$f = 2\left(\tfrac{1}{2} \times 2 - 3^2\right) = -16$$

$$h_1 := 3(2 + 4 \times 3) - 10 = 32$$

$$g_1 = -2 \times 2^2 + \tfrac{1}{3} \times 3 = -7$$

$$g_2 = -3 - 4 = -7$$

(ii) Evaluate gradients of the cost and constraint functions at the point (2,3).

$$\nabla f = \begin{bmatrix} x - y^2 \\ -2xy \end{bmatrix} = \begin{bmatrix} -7 \\ -12 \end{bmatrix}; \qquad \nabla h_1 = \begin{bmatrix} y \\ x + 8y \end{bmatrix} = \begin{bmatrix} 3 \\ 26 \end{bmatrix}$$

$$\nabla g_1 = \begin{bmatrix} -4x \\ 1/3 \end{bmatrix} = \begin{bmatrix} -8 \\ 1/3 \end{bmatrix}; \qquad \nabla g_2 = \begin{bmatrix} 0 \\ -1 \end{bmatrix}$$

(iii) Define the quadratic programming subproblem at the point (2,3).

Minimize

$$\bar{f} = -16 - 7d_1 - 12d_2 + \tfrac{1}{2}\left(d_1^2 + d_2^2\right)$$

subject to

$$32 + 3d_1 + 26d_2 = 0$$

$$-7 - 8d_1 + \tfrac{1}{3}d_2 \le 0$$

$$-7 - d_2 \le 0$$

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5. (8 points) An engineering design problem is formulated in terms of the design variables *x*, *y*, and *z* as follows:

Minimize $f = 2xz^2$

subject to

$$g_1: \ (4\times10^{-4})(5zy + 2z^2) - 1 \le 0$$

$$g_2: \ 1 - 2.5\times10^{-5}(x+y)z^2 \le 0$$

The quadratic programming subproblem has been defined and solved at the point (50, 10, 25). The Lagrange multipliers for the constraints are obtained as (20000, 70000). Using the initial value for the penalty parameter R_0 as 1, calculate the value of the descent function Φ at the given point.

$$f = 50\times25^2 = 31250$$

$$g_1 = (4\times10^{-4})(5\times25\times10 + 2\times25^2) - 1 = -0.9375$$

$$g_2 = 1 - 2.5\times10^{-5}(50+10)\times25^2 = 0.85$$

$$r = 20000 + 70000 = 90000$$

$$R = max(1{,}90000) = 90000$$

$$V = max\{0; -0.9375, 0.85\} = 0.85$$

$$\Phi = f + RV = 31250 + 0.85\times90000 = 107750$$