1. Vector of variables: \( x \in \mathbb{R}^n \)

2. Scalar function: \( f(x) \)

3. Vector function: \( g(x), x \in \mathbb{R}^n, g \in \mathbb{R}^m \)

4. **Gradient and Hessian of a function**: Gradient is a column vector of first partial derivatives of \( f \) with respect to \( x \), and Hessian is symmetric matrix of second partial derivatives:
   \[
   \nabla f(x), \text{ or } \frac{\partial f(x)}{\partial x} = \left\{ \frac{\partial f}{\partial x_i} \right\}_{n \times 1} ; \quad \text{Hessiam matrix: } \nabla^2 f = \left[ \frac{\partial^2 f}{\partial x_i \partial x_j} \right]_{n \times n}
   \]

5. **Gradient matrix** of a vector function: Let \( g_i \) be the \( i \)th component of vector \( g(x) \). Then according to the above definition, gradient vector for the \( i \)th function is written as
   \[
   \nabla g_i(x), \text{ or } \frac{\partial g_i(x)}{\partial x} = \left\{ \frac{\partial g_i}{\partial x_k} \right\}_{n \times 1}
   \]
   Let \( a^{(i)} = \nabla g_i \). Then the gradient matrix is given as
   \[
   \frac{\partial g}{\partial x} = \left[ a^{(1)} \quad a^{(2)} \quad \cdots \quad a^{(m)} \right]. \text{ Or, it can be written as matrix}
   \[
   A_{n \times m} = \left[ \frac{\partial g_j}{\partial x_i} \right] ; \text{ each column of A is the gradient of a component of } g(x)
   \]

6. Let \( f(x) \) and \( g(x) \) be \( m \) dimensional functions of \( x \in \mathbb{R}^n \). Then
   \[
   \frac{\partial}{\partial x} (f \cdot g) = \frac{\partial f}{\partial x} g + \frac{\partial g}{\partial x} f ; \text{ This is an } n \times 1 \text{ vector}
   \]

7. Let \( f(x) \) be the quadratic form
   \[
   f(x) = \frac{1}{2} x \cdot Ax ; \quad x \in \mathbb{R}^n ; \quad A_{n \times n}
   \]
   \[
   \frac{\partial f}{\partial x} = \frac{1}{2} \left( A + A^T \right) x \quad \text{Hessian matrix: } \frac{\partial^2 f}{\partial x^2} = \frac{1}{2} \left( A + A^T \right) \]