Chaos Theory

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When we see irregularity we cling to randomness and disorder for explanations. Why should this be so? Why is it that when the ubiquitous irregularity of engineering, physical, biological, and other systems are studied, it is assumed to be random and the whole vast machinery of probability and statistics is applied?

−William Ditto
Georgia Institute of Technology

What is Chaos?

• Nonlinearity: If it is linear it cannot be chaotic

• Deterministic: It has to be deterministic, rather than probabilistic. Every future state of the system follows from some specific current state.

• Sensitive to Initial Conditions: Small changes in behavior in the initial state can lead to radically different behavior in its final state.

• Sustained Irregularity: Hidden order exists within the seemingly random system over long time scales

Implications of Chaos

• Long term prediction of the system is impossible due to sensitive dependence on initial conditions

• Short term prediction highly effective with extremely good accuracy unlike probabilistic statistical methods

• Control of chaotic systems are obtainable if the nonlinearity is understood and accounted for.
Examples of Chaos

- Flow of water in rivers and streams

Examples of Chaos

- Heart rhythm, brain activity, breathing

Examples of Chaos

- Electronic Circuits

Examples of Chaos

- Wind, weather patterns
Current Applications of Chaos

- Engineering
- Vibration Control
- Stabilization of Circuits
- Power Girds
- Fluidized Beds
- Computers
  - Switching of packets in networks
  - Encryption
  - Control of chaotic robotic systems

Current Applications of Chaos

- Management
  - Economic forecasting
  - Restructuring
  - Market prediction and intervention
- Consumer Electronics
  - Washing machine stabilization
  - Air conditioner comfort control
  - Heating system comfort control
- Aerospace
  - Rockwell Collins Control Technologies

Brief History of Chaos

- 1963: Edward Lorenz discovers first strange attractor
- 1975: Li and Yorke introduce the term Chaos Theory
- 1976: Robert May shows chaotic population behavior
- 1980: Mandelbrot introduces fractal geometry
- 1990: Ott et al begin chaos control theory
- 1990: Pecora posits synchronization of chaotic system
- 2003: Kantz and Schreiber publish Nonlinear Time Series Analysis
- 2009: Lack of chaos in brain waves linked to epilepsy

Nonlinearity

- Linear Equations/Systems
  - The behavior of the resulting system subjected to a complex input can be described as a sum of responses to simpler inputs
- Non-linear systems have no such relation

\[ x = x + 1 \]
\[ y = x^2 - 3x + 1 \]
Nonlinearity
• Nonlinear Equations/Systems
  • A system whose output is not proportional to its input.
  • Cannot be written as a linear combination of components
  \[ x_{i+1} = a - x_i^2 + by_i \]
  \[ y_{i+1} = x_i \]

Determinism
• Random / Probabilistic does not depend on previous value
  • Coin Toss
  • Random Walk

• Chaotic Systems rely on the previous state to determine the future state
  • Weather patterns
  • Motion of celestial objects

Sensitivity to Initial Conditions
• Extremely small perturbations to the initial state of a system causes the system to radically change rapidly

\[ x_i = \begin{cases} 
1 & \text{if } s \leq a \\
\frac{1}{x_{i-1} - s} & \text{if } a < s < \alpha \\
1 & \text{if } s \geq \alpha 
\end{cases} \]

Edward Lorenz
• Computer simulation of weather at MIT in 1963
• Would record the values of the system at the end of each night.
• Would input the recorded values into the system as a starting point each morning.
• System would rapidly diverge from expected calculations due to the changes in the initial conditions
• Hypothesized the Butterfly Effect
Sensitivity to Initial Conditions

- Butterfly Effect: A butterfly flapping its wings in China can cause thunderstorms in Kansas a week later.

Attractors

- Sets to which a system may attend, possibly after some transient time.
  - Fixed Point Attractor: system attends to a single fixed point (Square Root iteratively pressed on calculator)
  - Periodic Attractor: system attends to a set of points repeated over and over (Sine wave)
  - Strange Attractor: system attends to a specific boundary with an infinite number of trajectories which are never repeated within this boundary

Fixed Point Attractor

- System attends to a single point after many iterations
  - "Frozen state" in NK Landscape theory
  - Population extinction
  - Iterative pressing of Square Root key on calculator

Periodic Attractor

- System repeatedly tends to a given set of points
  - Sine waves
  - Driven Pendulums
  - Clocks
Strange Attractors

- System is bounded by a set of points
- There exist an infinite number of trajectories within these boundaries
- The trajectories never repeat one another (otherwise the system would be periodic)
- Caused by stretching and folding of phase space

Characteristics of Strange Attractor

- Often are discovered in systems with high dimensional state spaces
- Are usually not visualized due to dimensional constraints
- Exhibit Self Affinity
- Can readily be reconstructed from any one of its parameters
- Most often have fractional dimensions

Self-Affinity

- Refers to an object which is fractal in nature and whose pieces are scaled by different amounts in each of the parameters directions
- The object is self-similar as we zoom in on specific regions of the original object
Attractor Reconstruction

- After calculation of the time lag and the embedding dimension (Lecture 2) the attractor can be reconstructed from any single parameter.

Example of Attractor

- Electrical Water Heater Tank Temp
  - Temperature sample every 1 minute
  - Standard household usage for family of 4
  - 4000 data points collected
Example of Attractor

Wind Turbine Attractor
- Wind Turbine Actual Power Data
  - For wind turbine that is consuming power (not running)
  - 4000 data points considered
  - Data Sampled at 10 second intervals with no filtering

Wind Turbine Attractor

Computer Chip Attractor
- Attractor discovered from more than 2900 variables
- Attractor showed that the manufacturing process was faulty (yields less than 1%)
Computer Chip Attractor
- Attractor detected faulty process and saved many millions of dollars.
- Process was not to be multimodal but analysis showed it was

Implications of Chaotic Attractors
- Systems previously considered stochastic can now be shown to be deterministic
- Systems can be shown to be bounded by some deterministic region
  - Specific attractors define specific behavior
    - Normal Heart beat is chaotic
    - Heart beat of people with Fibromyalgia and MS are less chaotic (study in process in Iowa)
    - Less chaos in brain activity linked to epilepsy
    - Prediction is facilitated through the attractor bounds

Self Organized Criticality
- Systems which are chaotic often exhibit power law behavior (dissipation) \( \frac{1}{f^\alpha} \)
- Systems exhibiting power law behavior are defined to organize themselves into critical states which must be relaxed for the system to continue to exist.
  - Earth Quakes
  - Volcanic Activity
  - Cascading Power Failures
  - System Faults in Chaotic Systems

Sand Pile Model
- Slowly drop grains of sand onto a table at exactly the same point each time.
- A pile of the sand grains will begin to grow at the drop location.
- As the sand pile grows the slope of the pile will reach an angle of repose and avalanches will begin to occur, spilling grains of sand onto neighboring locations.
- As the neighboring locations become full more avalanches occur.
- Soon, so many avalanches are occurring that the sand is flowing off of the table.
- The avalanches are the self organized critical system relaxing itself.
Cascading Power Failure Example
- Power Grid with Low Demand and High Disturbance Rate

Cascading Power Failure Example
- Power Grid with High Demand and Low Disturbance Rate

SOC of Wind Turbine Faults

Implications of SOC
- Chaotic systems may organize themselves to a critical point.
- Understanding of the criticality inherent in the system can assist in predictive fault detection.
- Determining a chaotic system to be critically self-organized allows for preventative mitigation techniques.
  - Reduce Grid Load while increasing transmission
  - Use fault prediction to perform preventative maintenance on equipment.
Determining Chaos

- Most books on chaos talk about infinite time scales
- We measure finite time scales, therefore we need to determine if a system is chaotic from a finite dataset.
- Time series data is best for determination of chaos.
- If the system is sensitively dependent upon time then it is most likely chaotic.

Chaotic Time Series?

- Perform any preprocessing as usual
  - Replacing missing values not recommended
  - Averaging the data is not recommended
- Create an ensemble of surrogate datasets from the original dataset.
  - Randomly arrange the time instances to ignore time.
  - Select surrogates from a similar distribution
- Measure each ensemble and the original dataset against some invariant measurement.
  - Kolmogorov Complexity
  - Correlation Dimension

Chaotic Times Series?

- If the invariant measurement is the same (or close) for each of the surrogates and the original dataset then the system is not chaotic.
- If the invariant measurement is the same (or close) for each surrogate but different than the original dataset (measured by some ε of difference) then the dataset exhibits deterministic chaos.
- This is due to the dependence on the previous states and there for the system is deterministically chaotic.

Lecture 2 Look Ahead

- Analysis of Non-linear Data and Attractor Reconstruction
  - Non-linear Noise Reduction
  - Calculation of Time Lag
  - Calculation of Embedding Dimension
  - Calculation of Maximal Lyapunov Exponent
  - Calculation of System Entropy
  - Non-Linear Prediction
  - Case Studies