PRODUCTION PLANNING AND SCHEDULING
Part 1

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Planning Hierarchy
• Forecasting
• Master Production Planning (Scheduling)
• Material Requirements Planning (MRP)
• Capacity Balancing
• Production Scheduling

MPS
MRP
Balancing
Scheduling

Forecasting
MPS
MRP
Balancing
Scheduling

MRP II (Manufacturing Resource Planning II)

ERP = MRP II + ...

History of ERP
• 1970’s MRP Material Requirements Planning
• 1980’s MRPII Manufacturing Resource Planning
• 1990’s ERP Enterprise Resource Planning (e.g., SAP system)

Master Production Schedule specifies Sequence and Quantity of Products (C)

EXAMPLE

<table>
<thead>
<tr>
<th>Jan</th>
<th>Feb</th>
<th>March</th>
</tr>
</thead>
<tbody>
<tr>
<td>200 C1</td>
<td>150 C7</td>
<td>180 C14</td>
</tr>
<tr>
<td>195 C4</td>
<td>160 C6</td>
<td>180 C12</td>
</tr>
<tr>
<td>385 C1</td>
<td>160 C6</td>
<td>670 C7</td>
</tr>
<tr>
<td></td>
<td>128 C17</td>
<td>230 C9</td>
</tr>
</tbody>
</table>

ERP systems are used from
• Automotive industry
to
• Pharmaceutical industry
The University of Iowa
Intelligent Systems Laboratory

MRP and ERP Systems

Backward (top down) generation of a production plan

Forward (push) implementation of the production plan

Note: Kanban systems are pull systems

EXAMPLE: Material Requirements Records for the Spider Climber

Merged Material Requirements for Aluminum Pipe

The Basic MRP (ERP) Record

"Arithmetic"

<table>
<thead>
<tr>
<th>Period</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gross requirements</td>
<td>10</td>
<td>40</td>
<td>10</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Scheduled receipts</td>
<td>50</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>On hand</td>
<td>4</td>
<td>44</td>
<td>44</td>
<td>4</td>
<td>-6</td>
</tr>
<tr>
<td>Planned order releases</td>
<td>50</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Lead time = 1 period
Lot size = 50
Safety stock = 4

Note: On hand should be >= Safety stock
**Question?**

<table>
<thead>
<tr>
<th>Period</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gross requirements</td>
<td>10</td>
<td>40</td>
<td>10</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Scheduled receipts</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>On hand</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Planned order releases</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Lead time = 1 period
Lot size = 50
Safety stock = 4

**What 50?**

What 50 do to the MRP record?

**The Answer**

<table>
<thead>
<tr>
<th>Period</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gross requirements</td>
<td>10</td>
<td>40</td>
<td>10</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Scheduled receipts</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>On hand (safety stock)</td>
<td>4</td>
<td>54</td>
<td>44</td>
<td>44</td>
<td>4</td>
</tr>
<tr>
<td>Planned order releases</td>
<td>50</td>
<td>50</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Lead time = 1 period
Lot size = 50
Safety stock = 4

Previously omitted
New order release

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**Explosion of Requirements for Subassembly S1 and Part P2**

<table>
<thead>
<tr>
<th>Period</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>S1 Gross requirements</td>
<td>10</td>
<td>40</td>
<td>10</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Scheduled receipts</td>
<td>50</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>On hand</td>
<td>54</td>
<td>44</td>
<td>44</td>
<td>44</td>
<td>4</td>
</tr>
<tr>
<td>Planned order releases</td>
<td>50</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lead time = 1 period</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lot size = 50</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Period</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>P2 Gross requirements</td>
<td>50</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Scheduled receipts</td>
<td>100</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>On hand</td>
<td>58</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Planned order releases</td>
<td>100</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lead time = 1 period</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lot size = 50</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

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**CAPACITY BALANCING**

- MPS
- MRP
- Balancing
- Scheduling

---

**8 hour period**

5 operations to be assigned to 2 machines

- Part 1: 2 operations
- Part 2: 1 operation
- Part 3: 2 operations

NOTE: Operation is a set of tasks (e.g., removal of machining features) of a part is performed on one machine

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**The result of capacity balancing**

<table>
<thead>
<tr>
<th>Machine 1</th>
<th>1</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Capacity</td>
<td>420</td>
<td>minutes</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Machine 2</th>
<th>2</th>
<th>5</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Capacity</td>
<td>480</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Question: Is this Gantt chart a feasible schedule?

Why Not?

An assignment only due to two conflicts

CAPACITY BALANCING MODELS

MODEL 1: No splitting of batches

Parameters
- \( I \): set of batches of operations to be processed
- \( J \): set of machines
- \( T_{ij} \): time of processing batch \( i \) on machine \( j \)
- \( C_{ij} \): cost of processing batch \( i \) on machine \( j \)
- \( b_j \): processing time available on machine \( j \) (capacity of machine \( j \))

Decision variable

\[
\begin{align*}
x_{ij} &= 1 & \text{if batch } i \text{ of operations is processed on machine } j, \quad j \in J \\
x_{ij} &= 0 & \text{otherwise}
\end{align*}
\]

Model 1: No splitting of batches

\[
\begin{align*}
\min \sum_{i \in I} \sum_{j \in J} C_{ij} x_{ij} & \quad \text{Min total processing cost} \\
\sum_{j \in J} x_{ij} &= 1 & \text{One batch per machine} \\
\sum_{i \in I} T_{ij} x_{ij} & \leq b_j & \text{Capacity constraint} \\
x_{ij} &= 0, 1 & \text{i.e., i \in I, j \in J} & \text{Integrality constraint}
\end{align*}
\]

Example

<table>
<thead>
<tr>
<th>Batch x Machine</th>
<th>Machine</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>5</td>
</tr>
<tr>
<td>4</td>
<td>5</td>
</tr>
</tbody>
</table>

Machining capacity \( b_j \) = \{21, 20, 42\}
Min \[ 4x_{11} + 7x_{12} + 7x_{13} + 1.5x_{21} + \ldots + 2x_{53} \]

For batch \( i = 1 \) \[ x_{11} + x_{12} + x_{13} = 1 \]
(row) \( i = 2 \) \[ x_{21} + x_{22} + x_{23} = 1 \]
\( i = 3 \) \[ x_{31} + x_{32} + x_{33} = 1 \]
\( i = 4 \) \[ x_{41} + x_{42} + x_{43} = 1 \]
\( i = 5 \) \[ x_{51} + x_{52} + x_{53} = 1 \]

For machine \( j = 1 \) \[ 3x_{11} + x_{12} + 4x_{13} + 1x_{21} = 21 \]
(column) \( j = 2 \) \[ 8x_{12} + x_{22} + 6x_{23} + 2x_{42} = 20 \]
\( j = 3 \) \[ 7x_{13} + 6x_{23} + 3x_{33} + 3x_{53} = 42 \]

\[ \sum_{t \in T} x_{ij} \leq b_j \]

\[ x_{ij} = \begin{cases} 1 & j \in J \\ 0 & \text{for } i = 1 \text{ to } 5, j = 1 \text{ to } 4 \end{cases} \]

Solution

\[ x_{11} = 1, x_{22} = 1, x_{31} = 1, x_{43} = 1, x_{53} = 1 \]

Machine 1: batches 1, 3
Machine 2: batch 2
Machine 3: batches 4, 5

Model 2: Limited Tool Magazine Capacity

No batch splitting

\[ k_i \] space occupied in a tool magazine by tools required for operation \( i \) at machine \( j \)
\( f_j \) capacity of the tool magazine on machine \( j \)
\( q_j \) penalty for using the tool magazine on machine \( j \)
\( Z_j \) upper limit on the number of tool magazines to be used on machine \( j \)
\( z_j \) number of tool magazines required on machine \( j \)

\[ \min \sum_{i \in I} \sum_{j \in J} C_{ij} y_{ij} + \eta_i f_i \]
\[ \sum_{i \in I} x_{ij} = 1, \quad i \in I \]
\[ \sum_{i \in I} \sum_{j \in J} x_{ij} \leq b_j, \quad j \in J \]
\[ \sum_{i \in I} \sum_{j \in J} x_{ij} \leq z_j, \quad j \in J \]
\[ \sum_{i \in I} \sum_{j \in J} y_{ij} \leq Z_j, \quad j \in J \]

Min total processing + tool magazine penalty cost
One batch per machine
Machine capacity constraint
Tool magazine capacity constraint
Integrality constraint
Integrality + bounding constraint

Model 3: Batch Splitting is Allowed

\[ t_{ij} \] processing time of each operation from batch \( i \) on machine \( j \)
\[ c_{ij} \] processing cost of an operation from batch \( i \) on machine \( j \)
\[ a_i \] required number of operations in batch \( i \) (the size of batch \( i \))
\[ y_{ij} \] number of operations of batch \( i \) to be processed on machine \( j \)

\[ \min \sum_{i \in I} \sum_{j \in J} c_{ij} y_{ij} \]
\[ \sum_{i \in I} x_{ij} = a_i, \quad i \in I \]
\[ \sum_{i \in I} \sum_{j \in J} t_{ij} y_{ij} \leq b_j, \quad j \in J \]
\[ y_{ij} \geq 0, \quad \text{integer } i \in I, j \in J \]

Min total processing cost
Required number of operations
Machine capacity constraint
Integrality constraint

Example

(1) number of operation types \( |I| = 10 \)
(2) number of machine types \( |J| = 3 \)
(3) matrix of machining times

\[
\begin{array}{ccc}
1 & 2 & 3 \\
1 & 29.1 & 24.5 & \infty \\
2 & 18.4 & 20.0 & \infty \\
3 & 31.2 & 28.0 & \infty \\
4 & \infty & 14.5 & 16.5 \\
5 & 24.5 & 22.0 & \infty \\
6 & 16.5 & 14.5 & 17.4 \\
7 & 8.5 & 6.4 & \infty \\
8 & 35.4 & \infty & 39.1 \\
9 & 19.4 & 18.1 & \infty \\
10 & 24.1 & 26.8 & \infty \\
\end{array}
\]

Batch - machine matrix
(4) vector of batch sizes
\[ [a_i] = [18, 17, 15, 14, 15, 20, 12, 18, 12, 16] \]
(5) vector of machine capacity
\[ [b_j] = [1800, 1000, 1500] \]

**Solution**
\[ y_{12} = 18, y_{21} = 17, y_{33} = 15, y_{42} = 4, y_{43} = 10, \]
\[ y_{51} = 9, y_{52} = 6, y_{62} = 20, y_{72} = 12, y_{81} = 18, \]
\[ y_{91} = 12, y_{10,1} = 16 \]

**Machine 1:** 17 operations (of type 2), 9(5), 18(8), 12(9), 16(10)
**Machine 2:** 18(1), 4(4), 6(5), 20(6), 12(7)
**Machine 3:** 15(3), 10(4)

**NOTE:** Operations 5 are processed on machines 1 and 2

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**Line Balancing**

Assignment of tasks to stations

**Task x station matrix**

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**What leads to more uniform utilization of machine capacity:**

- Capacity balancing with batch splitting, or
- Capacity balancing without batch splitting?

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**When the capacity loading Gantt chart would be equivalent to the schedule Gantt chart?**

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**Manufacturing Scheduling**

**MPS**

**MRP**

**Balancing**

**Scheduling**
Manufacturing Scheduling

Definition
Scheduling is the assignment of operations, jobs, tasks, etc. to resources in time.

Example: Two machine schedule

Makespan = 12

Parameters Used in Scheduling Models
\( t_{ij} \) = processing time of operation \( oi \) on machine \( M_j \)
\( ri \) = readiness of operation \( oi \) for processing, i.e., the time \( oi \)'s available for scheduling
\( di \) = due date, i.e., the promised delivery time of operation \( oi \)
\( wi \) = weight (priority), which expresses the relative urgency of operation \( oi \)

\( Ci \) = completion time of operations \( oi \)
\( Fi \) = flow time (the difference between completion time and readiness), \( Fi = Ci - ri \)
\( Li \) = lateness (the difference between completion time and due date), \( Li = Ci - di \)
\( Ti \) = tardiness, \( Ti = \max \{Ci - di, 0\} \)