LAYOUT OF MACHINES AND FACILITIES

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Outline

• LAYOUT EXAMPLES
• SINGLE-ROW MACHINE LAYOUT
• DOUBLE-ROW MACHINE LAYOUT
• MULTIROW FACILITY LAYOUT
• ALGORITHMS

Big Picture

Jobs: Facility Designer
Facility Planner

AutoCAD Software
http://www.autodesk.com/

LAYOUT EXAMPLES

MANUFACTURING FACILITY LAYOUT

ASSEMBLY AREA LAYOUT
LAYOUT OF MACHINES

Methods:

• Visual, e.g., using templates
• Computer based (often an environment for evaluating various layout alternatives)
• Model based

Visualization types:

• 2D
• 3D
• Virtual reality

Single-Row Machine Layout

Double-Row Machine Layout

Multi-row Layout

M1 M2 M3
M4 M4 M6
M7 M8
BASIC DATA

- UNIT DISTANCE TRAVEL COST ($/m)
- OR FREQUENCY OF TRIPS [f]

- DISTANCE [m]
- OR TRAVEL TIME [t]

OBJECTIVE

Arrange machines on the shop floor in such a way that the total product of the travel cost per unit distance traveled [$/m$] and the distance traveled [m] is minimum.

Travel cost is often replaced with the number (frequency - $f$) of trips per time unit [1/sec] between any two machines.

Distance is replaced with travel time - $t$ [sec].

\[
\text{Min } \sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{k=1}^{n} \sum_{l=1}^{n} f_{ik} t_{jl} x_{ij} x_{kl}
\]

Example: Intuitive Machine Layout

1
2
3
4
5

20 sq.m. 12 sq.m.
24 sq.m.
16 sq.m.
40 sq.m.

High Frequency of Traffic

SINGLE-ROW MACHINE LAYOUT

Frequency of trips (traffic flow)

\[
f_{ij} = \frac{n_{ij}}{u_k} \left\lceil \frac{v_{ij}}{k} \right\rceil
\]

where:

- $v_{ij} =$ volume of part type $k$ to be moved from machine $i$ to machine $j$ in a given time horizon (for example, 1 year)
- $n_{ij} =$ number of different parts to be moved from machine $i$ to machine $j$ in a given horizon
- $u_k =$ number of parts to be moved in a single trip of the material handling equipment
- $\lceil \cdot \rceil =$ smallest integer greater than or equal to $\cdot$

The AGV travel time involves:

- loading time
- acceleration time
- travel time
- deceleration time
- unloading time

DISTANCE CALCULATION

Consider 3 machines

\[
\text{Unidirectional distance (along x axis)} \quad r_{13} = 0, \quad r_{12} = 7, \quad r_{23} = r_{12} = 7
\]

\[
\text{Rectilinear (city) distance} \quad d_{13} = 3, \quad d_{12} = 7, \quad d_{23} = 7 + 3 = 10
\]

\[
\text{Euclidean distance} \quad e_{12} = 7, \quad e_{13} = 7.48
\]
Adjacent unidirectional distance $r_{12}$

\[
\begin{array}{c}
30 \\
M1 \\
36 \\
1 \\
20 \\
M2 \\
50
\end{array}
\]

\[r_{12} = d_{12} = 25 + 10 + 1 = 36\]

Time is the best measure to be used in the optimization of machine layout, however, if the travel speed is constant, an appropriate distance measure can be considered.

**SINGLE ROW MACHINE LAYOUT**

**Algorithm 1 Overview**

Two machines with the maximum flow between them (e.g., 1 and 2) are placed in the adjacent locations.

Other machines are placed to the left and right of the two machines based on the flow.

**Algorithm 1**

Step 0. Set iteration number $k = 1$.

From the flow matrix $[f_{ij}]$ compute $f_{i^*j^*} = \max \{f_{ij}: i, j = 1, 2, \ldots, m\}$. If there is a tie, $f_{i^*j^*} = \max \{f_{ij} \cdot t_{ij}: i, j = 1, 2, \ldots, m\}$. Connect $i^*$, $j^*$ and include them in the solution set. Set the solution set, $U = \{i^*, j^*\}$.

**Step 1.** Compute $f_{pq} = \max \{f_{pqr}, f_{qrs}, k, l \in \{1, 2, \ldots, m\} - U\}$.

Set $s^* = q^*$.

Consider two alternatives:

(a) Place machine $s^*$ left of machine $i^*$;
(b) Place machine $s^*$ right of machine $j^*$.

Compute:

$\sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{k=1}^{n} \sum_{l=1}^{n} f_{ijkl} x_{ijl}$

\[\text{Min} \ n \ n \ n \ n \ n\]

The minimization of the total product of flow $x$ travel time is accomplished by local maxima.

**Global Minimum**

**Local Maximum**
If \( q_{si} \leq q_{sj} \ast \)

1. Select the alternative (a) above;
2. Set \( i^* = s^* \).

If \( q_{si} > q_{sj} \ast \)

1. Select the alternative (b) above;
2. Set \( j^* = s^* \).

\[ U = U + s^* \]

Step 2. Set iteration number \( k = k + 1 \).
Repeat Step 1 until the final solution is obtained (i.e., until all the machines are included in the solution set \( U \)).

Example

Single-row machine layout

Given:

\[ [f_{ij}] = \begin{bmatrix} 
1 & 0 & 10 & 15 \\
2 & 10 & 0 & 5 \\
3 & 15 & 0 & 40 \\
4 & 15 & 5 & 40 \\
\end{bmatrix} \]

- \( f_{ij} \) = frequency of trips
- \( t_{ij} \) = travel time
- \( r_{ij} \) = unidirectional distance
- \( d_{ij} \) = adjacent distance

Clearance = 1

Calculate adjacent distances

\[ [d_{ij}] = \begin{bmatrix} 
1 & 0 & 4 & 5 & 3 \\
2 & 4 & 0 & 6 & 4 \\
3 & 5 & 6 & 0 & 5 \\
4 & 3 & 4 & 5 & 0 \\
\end{bmatrix} \]

For simplicity assume time \( t_{ij} = distance d_{ij} \)

Iteration 1

Step 0.
From the flow matrix \([f_{ij}]\),
\[ \max \{ f_{ij} : i = 1, 2, 3, 4, j = 1, 2, 3, 4 \} = f_{34} = 40 \]

is determined.
Machines 3 and 4 are connected and included in the solution, symbolically denoted as

\[ 34 \]

The solution set is updated \( U = \{3, 4\} \) and the columns 3 and 4 of the matrices are marked with asterisks.

Distances

\[ \begin{array}{c}
50 \\
30 \\
\end{array} \]

\[ \begin{array}{c}
1 \\
20 \\
\end{array} \]

36

Adjacent distance \( d_{12} = 25 + 10 + 1 = 36 \)

Unidirectional distance \( r_{12} = d_{12} = 36 \)
Step 1. Compute
\[ \text{Max} \{ f_{3k}, f_{4l} : k = 1, 2; l = 1, 2 \} = f_{31} = f_{41} = 15. \]
Set \( s^* = 1. \)
Consider two alternatives:
(a) Place machine 1 on the left hand side of machine 3
(b) Place machine 1 on the right hand side of machine 4

\[
\begin{array}{cccc}
1 & 2 & 3 & 4 \\
1 & 0 & 10 & 15 & 15 \\
2 & 10 & 0 & 0 & 40 \\
3 & 15 & 5 & 40 & 0 \\
4 & 15 & 0 & 0 & 0 \\
\end{array}
\]

\[ q_{13} = f_{13} \cdot t_{13} + f_{14} \cdot t_{14} + f_{11} \cdot t_{11} = f_{13} \cdot r_{13} + f_{14} \cdot r_{14} + f_{11} \cdot r_{11} \\
= f_{13} \cdot d_{13} + f_{14} \cdot d_{14} + f_{11} \cdot d_{11} = 15 \cdot 3 + 15 \cdot (3 + 5) = 165. \]

As \( q_{13} > q_{41}, \)
(1) Alternative (b) \( \text{[min]} \) is selected where machine 1 is placed to the right of machine 4
(b) Place machine 1 on the right hand side of machine 4

Step 2. Since machine 2 is not included in the solution set \( U, \)
go to Step 1.

Compute (repeated from the previous slide):
\[ q_{23} = q_{23} = f_{23} \cdot t_{23} + f_{24} \cdot t_{24} + f_{21} \cdot t_{21} = f_{23} \cdot r_{23} + f_{24} \cdot r_{24} + f_{21} \cdot r_{21} \\
= f_{23} \cdot d_{23} + f_{24} \cdot d_{24} + f_{21} \cdot d_{21} = 10 \cdot 4 + 5 \cdot (4 + 3) + 0 \cdot (4 + 3 + 5) = 195. \]

Since \( q_{23} > q_{12}, \)
(1) Alternative (b) \( \text{[min]} \) is selected, i.e.,
(2) Row 1 and column 1 are deleted from \( [f_{ij}] \);
(3) Set \( j^* = 2; \)
(4) The solution set \( U \) is updated to \( U + s^* = \{1, 2, 3, 4\}. \)

Step 2. Since all machines have been included in the solution set \( U, \) stop.
Algorithm 1 Summary

Pair (3, 4) selected

Look around

Machine 1 selected

Machine 1 placed

Algorithm 1 Summary

Alternative selected

Look around

Machine 2 selected

Machine 2 placed

Objective Function Revisited

Min \( \sum_{ijkl} f_{ijkl} x_{ij} x_{kl} \)

\( f \rightarrow \) No trips/sec

\( t \rightarrow \) sec

\( f \cdot t = \) No of trips

Double-Row Machine Layout

Example Machine Layout
Computation of Unidirectional Distances Between Pairs of Machines

Unidirectional distances

\[ r_{12} = 0 \quad r_{14} = d_{24} \]
\[ r_{13} = d_{13} \quad r_{23} = d_{13} \]
\[ r_{24} = d_{24} \quad r_{34} = |r_{13} - r_{24}| = |d_{13} - d_{24}| \]

Algorithm 2: Double-row machine layout

- A pair of machines with the largest value of the flow is selected
- The machines selected are placed on the opposite sites of the isle (or an AGV track)
- Other pairs of machines are placed to the left and right of the pairs selected.

Compute:

- \( q_i^s v_j^* t_t^* = \sum q_i^s v_j^* t_t^* \)
- \( q_i^s v_j^* t_t^* \)

Step 2. If only one machine has not been assigned, go to Step 4.
Otherwise go to Step 3.
Compute:

\[ q_c^* d^* j^* = \{ f_c r \cdot t_c r + f_d r \cdot t_d r : r \in U \} + f_c d^* \cdot t_c d^* , \]

where \( c^* \) is placed left of \( i^* \) and \( d^* \) is placed left of \( j^* \);

\[ q_d^* i^* c^*_j^* = \{ f_c r \cdot t_c r + f_d r \cdot t_d r : r \in U \} + f_c d^* \cdot t_c d^* , \]

where \( d^* \) is placed left of \( i^* \) and \( c^* \) is placed left of \( j^* \);

\[ q_s^* c^*_t^* d^* = \{ f_c r \cdot t_c r + f_d r \cdot t_d r : r \in U \} + f_c d^* \cdot t_c d^* , \]

where \( c^* \) is placed right of \( s^* \) and \( d^* \) is placed right of \( t^* \);

\[ q_s^* t^* c^*_d^* = \{ f_c r \cdot t_c r + f_d r \cdot t_d r : r \in U \} + f_c d^* \cdot t_c d^* , \]

where \( d^* \) is placed right of \( s^* \) and \( c^* \) is placed right of \( t^* \).

\[ \sum \sum \sum \sum \sum \]

If \( q_c^* d^* j^* = \min \{ q_c^* d^* j^* , q_d^* i^* c^*_j^* , q_s^* c^*_t^* d^* , q_s^* t^* c^*_d^* \} , \)
(1) select alternative (a);
(2) set \( i^* = c^* \) and \( j^* = d^* \).

If \( q_d^* i^* c^*_j^* = \min \{ q_c^* d^* j^* , q_d^* i^* c^*_j^* , q_s^* c^*_t^* d^* , q_s^* t^* c^*_d^* \} , \)
(1) select alternative (b);
(2) set \( i^* = d^* \) and \( j^* = c^* \).

If \( q_s^* c^*_t^* d^* = \min \{ q_c^* d^* j^* , q_d^* i^* c^*_j^* , q_s^* c^*_t^* d^* , q_s^* t^* c^*_d^* \} , \)
(1) select alternative (c);
(2) set \( s^* = c^* \) and \( t^* = d^* \).

If \( q_s^* t^* c^*_d^* = \min \{ q_c^* d^* j^* , q_d^* i^* c^*_j^* , q_s^* c^*_t^* d^* , q_s^* t^* c^*_d^* \} , \)
(1) select alternative (d);
(2) set \( s^* = d^* \) and \( t^* = c^* \).

Set \( U = U + \{ c^* , d^* \} \).
Go to Step 5.

Step 4. Set \( c^* \) to the last machine which has not been assigned.
Consider four alternatives:
(a) Place \( c^* \) left of \( i^* \);
(b) Place \( c^* \) left of \( j^* \);
(c) Place \( c^* \) right of \( s^* \);
(d) Place \( c^* \) right of \( t^* \).
Compute:

\[ q_c^* i^* = \{ f_c r \cdot t_c r : r \in U \} \]

where \( c^* \) is placed left of \( i^* \);

\[ q_c^* j^* = \{ f_c r \cdot t_c r : r \in U \} \]

where \( c^* \) is placed left of \( j^* \);

\[ q_s^* c^*_ = \{ f_c r \cdot t_c r : r \in U \} \]

where \( c^* \) is placed right of \( s^* \);

\[ q_t^* c^*_ = \{ f_c r \cdot t_c r : r \in U \} \]

where \( c^* \) is placed right of \( t^* \).

\[ \sum \sum \sum \sum \sum \]

If \( q_c^* i^* = \min \{ q_c^* i^* , q_c^* j^* , q_s^* c^*_ , q_t^* c^*_ \} , \)
select alternative (a);
If \( q_c^* j^* = \min \{ q_c^* i^* , q_c^* j^* , q_s^* c^*_ , q_t^* c^*_ \} , \)
select alternative (b);
If \( q_s^* c^*_ = \min \{ q_c^* i^* , q_c^* j^* , q_s^* c^*_ , q_t^* c^*_ \} , \)
select alternative (c);
If \( q_t^* c^*_ = \min \{ q_c^* i^* , q_c^* j^* , q_s^* c^*_ , q_t^* c^*_ \} , \)
select alternative (d).
Set \( U = U + \{ c^* \} \).
Go to Step 5.

Step 5. Set iteration number \( k = k + 1 \).
Repeat Steps 2 through 4 until the final solution is obtained (i.e., all the machines are included in the solution set \( U \)).

Distances

Distance \( d_{ij} \) is calculated from machine dimensions and clearance.

Based on \( d_{ij} , r_{ij} \) is computed in the algorithm.
Assume \( t_{ij} = r_{ij} \).
Recall?

Adjacent distance \( d_{12} = 25 + 10 + 1 = 36 \)

Unidirectional distance \( r_{12} = d_{12} = 36 \)

Step 1. For the flow matrix above, compute max \( \{f_{ij}: i, j = 1, 2, ..., 5\} = f_{12} = f_{45} = 5 \) is determined. The flow value \( f_{45} \) is selected because \( f_{45} \cdot t_{45} = f_{45} \cdot d_{45} = 5 \cdot 48 = 240 > f_{12} \cdot t_{12} = f_{12} \cdot d_{12} = 5 \cdot 36 = 180 \).

Thus \( i^* = 4 \) and \( j^* = 5 \).

Machines 4 and 5 are assigned to the opposite sites of the AGV path and are included in the solution.

The solution set is updated, \( U = \{4, 5\} \). Exclude columns 4 and 5 from further consideration.

Computing:

max \( \{f_{5l}, f_{1v}: l, v = 2, 3\} = f_{12} \).

Set \( t^* = 2 \) and exclude column 2 from further consideration.

Consider two alternatives:

(a) Place machine 1 right of machine 4 and machine 2 right of machine 5

(b) Place machine 2 right of machine 4 and machine 1 right of machine 5

Distance Pattern

\( q_{s^*i^*t^*j^*} = q_{4152} = f_{14} + t_{14} + t_{15} + t_{15} + t_{24} + t_{24} + t_{25} + t_{25} + t_{12} + t_{12} = f_{14} + t_{14} + t_{15} + t_{24} + t_{25} + t_{25} + t_{12} + t_{12} + t_{41} + t_{41} + t_{51} + t_{51} + t_{42} + t_{42} + t_{52} + t_{52} + t_{45} + t_{45} + t_{54} + t_{54} + t_{41} + t_{41} + t_{52} + t_{52} + t_{45} + t_{45} + t_{26} + t_{26} + t_{53} = 482 \).
\[ q^{*}(4,5) = q_{425} = f_{24} \cdot r_{24} + f_{25} \cdot r_{25} + f_{14} \cdot r_{14} + f_{15} \cdot r_{15} + f_{12} \cdot r_{12} + f_{24} \cdot d_{24} + f_{25} \cdot d_{25} + f_{14} \cdot d_{14} + f_{15} \cdot d_{15} + f_{12} \cdot d_{12} = 0 \cdot 41 + 2 \cdot 41 + 4 \cdot 42 + 1 \cdot 42 + 5 \cdot |d_{24} - d_{15}| = 297. \]

Since \( q_{425} < q_{4152} \), alternative (b) is selected. Set \( U = U + \{1, 2\} \).

Step 2. Since only machine 3 has not been assigned, go to Step 4.

**Compute**

(a) \[ q^{*}(4,5) = q_{34} = f_{34} \cdot r_{34} + f_{35} \cdot r_{35} + f_{32} \cdot r_{32} + f_{31} \cdot r_{31} = f_{34} \cdot r_{34} + f_{35} \cdot r_{35} + f_{32} \cdot r_{32} + f_{31} \cdot r_{31} = 0 \cdot 43.5 + 0 \cdot 43.5 + 3 \cdot (43.5 + 41) + 1 \cdot (43.5 + 42) = 109.5 + 67 \cdot 3 = 339. \]

(b) \[ q^{*}(3,5) = q_{35} = f_{35} \cdot r_{35} + f_{34} \cdot r_{34} + f_{32} \cdot r_{32} + f_{31} \cdot r_{31} = f_{35} \cdot r_{35} + f_{34} \cdot r_{34} + f_{32} \cdot r_{32} + f_{31} \cdot r_{31} = 0 \cdot 28.5 + 0 \cdot 28.5 + 3 \cdot (28.5 + 41) + 1 \cdot (28.5 + 42) = 279. \]

Since \( q_{35} = \min \{q_{34}, q_{35}, q_{23}, q_{13}\} \), alternative (c) is selected. Set \( U = U + \{3\} \).

Step 5. Since all machines have been included in the set \( U \), stop.

**Solution**

- Machine 1
- Machine 2
- Machine 3
- AGV
- Reference line

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**Note:** The text is from a document discussing the selection of alternatives in a planning or scheduling context, specifically focusing on the calculation of certain metrics and the selection of the best alternative based on those calculations.
Algorithm 2 Summary

Pair (4, 5) selected

Looking around

Machine 1 selected

Looking for

Machine 2 selected

Evaluate cost

a) Alternative (b) selected

b) Alternative (b) selected

Lowest cost alternative selected

MULTIROW FACILITY AND MACHINE LAYOUT

Quadratic Assignment Model

\[ \text{Min} \quad n \sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{k=1}^{n} f_{ik} c_{jl} x_{ij} x_{kl} + \Sigma a_{ij} x_{ij} \]

subject to:

\[ \sum_{j=1}^{n} x_{ij} = 1 \quad i = 1, ..., n \]

\[ \sum_{i=1}^{n} x_{ij} = 1 \quad j = 1, ..., n \]

\[ x_{ij} = 0 \text{ or } 1 \quad i = 1, ..., n, \quad j = 1, ..., n \]
MULTIROW FACILITY(MACHINE) LAYOUT

CRAFT Algorithm

FLOW MATRIX (between facilities (machines))

Assumptions:
- Symmetric flow matrix
- Square facilities

Matrix of Rectilinear Distances (between sites)

1  2  3  4
A  | 1 - 50  20  100 |
B  | 50  - 30  10  |
C  | 20  30  - 70  |
D  | 100 10  70  - |

Matrix of Rectilinear Distances (between sites)

1  2  3  4
A  | 1  1  2  3 |
B  | 1  - 1  2  |
C  | 2  1  - 1  |
D  | 3  2  1  - |

PAIRWISE EXCHANGES

First exchange: Machines (1, 2)

Cost = f12dAD + f13dBD + f14dCD + f23dAB + f24dAC + f34dBC
= 50×3 + 20×2 + 100×1 + 30×1 + 10×2 + 70×1 = 410.

COST = 510, A • 1 B • 3 C • 4 = D • 2

Exchange facilities (machines) 1 and 2

Cost = 410, A • 2 B • 3 C • 4 = D • 1

The cost change is then 410 - 510 = -100.
The Result

Exchanged Pair
(1, 2) (1, 3) (1, 4) (2, 3) (2, 4) (3, 4)

Cost Change
-100 -50 -80 -60 90 -100

A  2 B  3 C  4
D  1

Cost 410
A  2 B  3 C  4 Exchange Pair (1, 2) (1, 3) (1, 4) (2, 3) (2, 4) (3, 4)
D  +1
Cost Change 100 10 30 100 100

Solution with facilities (machines) 1 and 3 exchanged is selected

Cost 370
A  2 B  1 C  4 Exchange Pair (1, 2) (1, 3) (1, 4) (2, 3) (2, 4) (3, 4)
D  3
Cost Change 80 40 90 90 40 60

Final Solution

Cost 370
A  2 B  1 C  4 Exchange Pair (1, 2) (1, 3) (1, 4) (2, 3) (2, 4) (3, 4)
D  3

Machine Relationship Constraints

This “box” shows the relationship between machine (facility) 1 and 3
Top half shows importance of relationship
Lower half shows reason(s) of importance
“Closeness” ratings

EXAMPLE: Flow data and relationship constraints

Process Layout: Medical Application
Data Collection

Process 1

a
Station 1
M1 O1

b
Station 2

O2

c
Resources

Possible Groupings

Possible Groupings

Station

Note: Precedences are ignored in this matrix

Process 1

Possible Groupings

Station

Process

Possible Groupings

Based on the process-station grouping

System Layout

Process 1

Process 2