Five batches of parts are to be processed in a manufacturing system with three machines. Given the matrix of batch processing costs \([C_{ij}]\), the matrix \([T_{ij}]\) of processing of each batch on the three machines, the machine capacity vector \([b_j]\), the matrix \([k_{ij}]\) of space occupied by tools in tool magazines, the tool magazine capacity vector \([f_j]\), the penalty vector \([q_j]\), and the vector \([Z_j]\) of upper limits on the number of tool magazines to be used on machine \(j\).

\[ [C_{ij}] = \begin{bmatrix} 4 & 6 & 5 \\ 2 & 3 & 6 \\ 3 & 8 & 5 \\ 4 & 5 & 4 \\ 5 & 7 & 3 \\ 9 & 7 & 2 \end{bmatrix}, \quad [T_{ij}] = \begin{bmatrix} 1 & 2 & 3 \\ 6 & 9 & 7 \\ 2 & 2 & 1 \end{bmatrix}, \quad [b_j] = [15, 15, 15]^T \]

Assign optimally the parts and tools to the machines. What is the number of tool magazines used on each of the three machines?

**Solution**

**Integer Programming Program**

\[
\begin{align*}
\text{Min} & \quad 4x_{11} + 6x_{12} + 5x_{13} + 2x_{21} + 3x_{22} + 6x_{23} + 3x_{31} + 8x_{32} + 5x_{33} + 4x_{41} + 5x_{42} + 4x_{43} + 7x_{51} + 3x_{52} + 9x_{53} + 1000(z_1 - 1) + 1000(z_2 - 1) + 1000(z_3 - 1) \\
x_{ij} & = 1 \text{ if batch } i \text{ processed on machine } j \\
x_{ij} & = 0 \text{ otherwise} \\
x_{11} + x_{12} + x_{13} = 1 \\
x_{21} + x_{22} + x_{23} = 1 \\
x_{31} + x_{32} + x_{33} = 1 \\
x_{41} + x_{42} + x_{43} = 1 \\
x_{51} + x_{52} + x_{53} = 1 \\
6x_{11} + 2x_{21} + 4x_{31} + 4x_{41} + 2x_{51} \leq 15 \\
9x_{12} + 1x_{22} + 3x_{32} + 7x_{42} + 3x_{52} \leq 15
\end{align*}
\]

One batch assigned to one machine only

Machine capacity

Limit on the No of tool magazines

INTEGER 15
GINT \(z_1\)
GINT \(z_2\)
GINT \(z_3\)

END
### IP Solution

**OBJECTIVE FUNCTION VALUE**

1) 17.000

<table>
<thead>
<tr>
<th>VARIABLE</th>
<th>VALUE</th>
<th>REDUCED COST</th>
</tr>
</thead>
<tbody>
<tr>
<td>X11</td>
<td>1.000000</td>
<td>4.000000</td>
</tr>
<tr>
<td>X12</td>
<td>0.000000</td>
<td>6.000000</td>
</tr>
<tr>
<td>X13</td>
<td>0.000000</td>
<td>5.000000</td>
</tr>
<tr>
<td>X21</td>
<td>1.000000</td>
<td>2.000000</td>
</tr>
<tr>
<td>X22</td>
<td>0.000000</td>
<td>3.000000</td>
</tr>
<tr>
<td>X23</td>
<td>0.000000</td>
<td>6.000000</td>
</tr>
<tr>
<td>X31</td>
<td>1.000000</td>
<td>3.000000</td>
</tr>
<tr>
<td>X32</td>
<td>0.000000</td>
<td>8.000000</td>
</tr>
</tbody>
</table>

According to this solution, one of each tool magazines is required.

\[ X_{11} = X_{21} = X_{31} = X_{43} = X_{52} = 1 \]

and \[ Z_1 = Z_2 = Z_3 = 1 \]

### Solution

\[
\begin{bmatrix}
1 & 2 & 3 \\
4 & 6 & 5 \\
2 & 3 & 6 \\
3 & 8 & 5 \\
4 & 4 & 5 & 4 \\
5 & 7 & 3 & 9
\end{bmatrix} = \begin{bmatrix} b_i \end{bmatrix} = [15, 15, 15]
\]

\[
\begin{bmatrix}
1 & 2 & 3 \\
6 & 9 & 7 \\
2 & 1 & 5 \\
4 & 3 & 2 \\
4 & 4 & 7 & 5 \\
5 & 2 & 3 & 1
\end{bmatrix}
\]