SELECTION OF MANUFACTURING EQUIPMENT

Andrew Kusiak
Intelligent Systems Laboratory
2139 Seamans Center
The University of Iowa
Iowa City, Iowa 52242 - 1527
Tel: 319 - 335 5934    Fax: 319-335 5669
andrew-kusiak@uiowa.edu
http://www.icaen.uiowa.edu/~ankusiak

Steps in design of manufacturing systems
- Equipment selection
- Machine cell formation
- Machine layout
- Cell layout

Equipment Selection
Machine Cell Formation
Machine Layout
Cell Layout

OR

Equipment Selection
Machine Layout

Manufacturing Equipment Selection
The results of selecting the right type and number of manufacturing equipment:
- Reduced procurement cost
- Reduced operating and maintenance cost
- Increased machine utilization
- Improved layout of equipment
- Increased efficiency of a manufacturing system

Layout of Machines
Example: Layout Patterns
Articulated robot (R)       AGV

Model 1: Selection of machines

\[ M_i = \frac{\sum t_j}{h \cdot u} \]

where:
- \( M_i \) the number of machines of type \( i \)
- \( t_j \) processing time of part \( j \) [hours]
- \( p \) number of parts
- \( h \) number of standard hours/day
- \( u \) scrap factor?
Model 2: Example 1
Matrix defining relationship between operations and machines
\[
\begin{bmatrix}
1 & 1 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 & 1 \\
\end{bmatrix}
\]
\[
[a_{ij}] =
\begin{bmatrix}
0.5 & 0.6 & 0.4 & 0.95 & 1.1 & 0.42
\end{bmatrix}
\]

Model 2: Selection of machines
\[
\begin{align*}
\min \sum_{j=1}^{5} x_j & \\
\sum_{j=1}^{6} a_{ij} x_j & \geq 1 \quad i = 1, \ldots, 5 \\
x_j & = 0, 1 \quad j = 1, \ldots, 6
\end{align*}
\]
Solution: \(x_4 = x_5 = 1\), i.e., machines M4 and M5, with the total processing time of 2.05

Selection of Machines and Material Handling Systems
Generalized operation-manufacturing equipment incidence matrix
\[
\begin{bmatrix}
R_1 = M_1 + H_1 & R_2 = M_2 + H_2 & R_3 = M_1 + M_2 + H_3 & R_4 = M_4 + H_4 & R_5 = M_5 + H_5 & R_6 = M_6 + H_6 & R_7 = M_6 + M_7 + H_7 \\
\end{bmatrix}
\]
\[
[a_{ij}] =
\begin{bmatrix}
1 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 \\
\end{bmatrix}
\]

Example 2
Given:
1. Vector O of operations:
   \(O = \{o_1, o_2, o_3, o_4, o_5, o_6\}\)

2. Vector R of modules:
   \(R = \{R_1 = M_1 + H_1, R_2 = M_2 + H_2, R_3 = M_1 + M_2 + H_3, R_4 = M_4 + H_4, R_5 = M_5 + H_5, R_6 = M_6 + H_6, R_7 = M_6 + M_7 + H_7\}\)

3. Vector C of costs of modules:
   \(C = \{c_1, c_2, c_3, c_4, c_5, c_6, c_7\}\)

Model 3: Selection of machines and MHSs
\[
\begin{align*}
\min \sum_{j=1}^{n} c_j x_j \\
\sum_{j=1}^{n} a_{ij} x_j & \geq n_i \quad \text{for all } i = 1, \ldots, m \\
\sum_{j=1}^{n} x_j & \leq p_k \quad \text{for all } k = 1, \ldots, p \\
x_j & = 0, 1 \quad \text{for all } j = 1, \ldots, n
\end{align*}
\]
4. Vector of the required number of machines that can perform operation $o_i:\n = [n_i] = [2, 2, 1, 2, 2, 1]^T$

5. Generalized operation - module equipment incidence matrix

\[
\begin{array}{cccc}
F1 & F2 & F3 & F4 \\
1 & M_1 & M_2 & M_2 & M_2 & M_2 & M_2 \\
2 & 3 & H_1 & H_1 & H_1 & H_1 & H_1 \\
3 & 4 & M_2 & H_2 & H_2 & H_2 & H_2 \\
4 & 6 & 5 & H_2 & H_2 & H_2 & H_2 \\
5 & 7 & 6 & M_2 & H_2 & H_2 & H_2 \\
6 & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\
7 & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\
\end{array}
\]

\[
\begin{array}{cccc}
1 & 1 & 1 & 1 \\
2 & 1 & 1 & 1 \\
3 & 1 & 1 & 1 \\
4 & 1 & 1 & 1 \\
5 & 1 & 1 & 1 \\
6 & 1 & 1 & 1 \\
7 & 1 & 1 & 1 \\
\end{array}
\]

\[
\begin{array}{cccc}
150,000 & 130,000 & 220,000 & 190,000 \\
90,000 & 110,000 & 210,000 & 0 \\
\end{array}
\]

**Input file**

\[
\begin{align*}
\text{MIN } & 150 X_1 + 130 X_2 + 220 X_3 + 190 X_4 + 90 X_5 + 110 X_6 + 210 X_7 \\
\text{SUBJECT TO } & \\
& 1X_1 + 1X_2 + 2X_3 + 1X_4 + 1X_5 + 1X_6 + 1X_7 \geq 2 \\
& 1X_1 + 1X_2 + 3X_3 + 1X_4 \geq 1 \\
& 1X_1 + 1X_3 + 1X_4 + 1X_6 + 1X_7 \geq 2 \\
& 1X_5 + 1X_6 + 2X_7 \geq 2 \\
& 1X_6 + 2X_7 \geq 1 \\
& 1X_1 + 1X_2 + 1X_3 + 1X_4 + 1X_6 + 1X_7 \leq 2 \\
& 1X_5 + 1X_6 + 1X_7 \leq 1 \\
& 1X_1 + 1X_2 + 1X_3 \leq 1 \\
& 1X_4 \leq 1 \\
& 1X_5 \leq 1 \\
& 1X_6 + 1X_7 \leq 1 \\
\text{END} \\
\end{align*}
\]

**Integer 7**

**Special Case of the Equipment Selection Model**

In some cases, purchasing decision regarding only machines or material handling equipment is made.

**Model**

\[
\begin{align*}
\text{Min } & \sum_{j=1}^{n} c_{j} x_{j} \\
\text{subject to } & \sum_{j=1}^{n} a_{ij} x_{j} \geq n_{i} \text{ for all } i = 1, \ldots, m \\
& x_{j} = 0, 1 \text{ for all } j = 1, \ldots, n
\end{align*}
\]

**Solution:** $x_3=1, x_5=1, x_6=1$

Machines selected: M1, M2, M5, M6


The total cost of the equipment is $420,000.

**SELECTION OF MANUFACTURING RESOURCES BASED ON PROCESS PLANS**

The manufacturing resources of concern here are:

- Machines
- Tools to machine or assemble parts,
- Fixtures for holding parts to be machined or assembled,
- Grippers for handling parts, and
- Feeders for presenting parts

\[
\begin{align*}
\text{Feeders } e_1 \ldots e_7 \\
\text{Tools } t_1 \ldots t_{10} \\
\text{Part } \quad \text{Grippers } g_1 \ldots g_10
\end{align*}
\]
Three reasons for solving the resource selection problem:

• Reduction of manufacturing cost
• Limited capacity of tool magazines
• Reduction of the number of types of machines and auxiliary devices

Added cost

No of different types of resources

Example 3
Consider the vector of weight coefficients \([w] = \begin{bmatrix} 1 \\ 2 \\ 4 \\ 1 \\ 3 \\ 5 \end{bmatrix}\) and two process plans \(PP_1\) and \(PP_5\)

\[ [w] = \begin{bmatrix} 1 \\ 2 \\ 4 \\ 1 \\ 3 \\ 5 \end{bmatrix} \]
\[ PP_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \]
\[ PP_2 = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} \]

Hamming distance
\[ d_{15} = 1 + 0 + 1 + 1 + 1 + 0 = 4 \]

The weighted Hamming distance
\[ d_{15} = 1x1 + 2x0 + 4x1 + 1x1 + 3x1 + 5x0 = 9 \]

Resource Selection Model

**Notation**

\( K = \{ 1, 2, ..., m \} \) is the set of parts to be manufactured

\( N_k \) is the set of process plans, where

\( k \in K \)

\( A \) is the set of arcs connecting process plans from set \( N_k \) to set \( N_l \) \( k \neq l \)

\( c_i \) is the cost of process plan \( PP_i \), for all \( i \in N \)

\[ xi = \begin{cases} 1 & \text{if process plan } PP_i \text{ is selected} \\ 0 & \text{otherwise} \end{cases} \]

\[ y_{ij} = \begin{cases} 1 & \text{if process plan } PP_i \text{ and } PP_j \text{ is selected} \\ 0 & \text{otherwise} \end{cases} \]

\[ xi = \begin{cases} 1 & \text{if process plan } PP_i \text{ is selected} \\ 0 & \text{otherwise} \end{cases} \]

**Model**

\[ \text{min } \frac{1}{2} \sum_{(i,j) \in A} d_{ij} y_{ij} + \sum_{i \in N} c_i x_i \]

subject to:

\[ \sum_{i \in N_k} x_i = 1 \quad k \in K \]

\[ x_i + x_j - 1 \leq y_{ij} \quad (i, j) \in A \]

\[ x_i = 0, 1 \quad i \in N \]

\[ y_{ij} = 0, 1 \quad (i, j) \in A \]

Lean Mfg Model

\[ \text{min } \frac{1}{2} \sum_{(i,j) \in A} d_{ij} y_{ij} + \sum_{i \in N} c_i x_i \]

subject to:

\[ \sum_{i \in N_k} x_i = 1 \quad k \in K \]

\[ x_i + x_j - 1 \leq y_{ij} \quad (i, j) \in A \]

\[ x_i = 0, 1 \quad i \in N \]

\[ y_{ij} = 0, 1 \quad (i, j) \in A \]

Lean Mfg Model

\[ \text{Relaxing the integrality of the constraint transforms model into the following mixed integer programming program:} \]

\[ \text{Min } \frac{1}{2} \sum_{(i,j) \in A} d_{ij} y_{ij} + \sum_{i \in N} c_i x_i \]

Subject to:

\[ \sum_{i \in N_k} x_i = 1 \quad k \in K \]

\[ x_i + x_j - 1 \leq y_{ij} \quad (i, j) \in A \]

\[ x_i = 0, 1 \quad i \in N \]

\[ y_{ij} = 0, 1 \quad (i, j) \in A \]
Example 4
Consider the following data sets:
1. Set of parts \( K = \{1, 2, 3, 4\} \).
2. Sets of process plans \( N_1 = \{1, 2\} \), \( N_2 = \{3, 4, 5\} \), \( N_3 = \{6, 7\} \), \( N_4 = \{8, 9, 10\} \).
3. The incidence matrix for five tools, three fixtures, and four parts (10 process plans):

<table>
<thead>
<tr>
<th>Tools</th>
<th>Part 1</th>
<th>Part 2</th>
<th>Part 3</th>
<th>Part 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>T1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>T2</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>T3</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>T4</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>T5</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

4. Vector of process plan costs:
\[ \mathbf{c} = [5.8, 9.4, 11.6, 5.7, 3.4, 4.3, 5.1, 6.4, 5.2, 5.3] \]
5. Vector of weights:
\[ \mathbf{w} = [1, 1, 1, 1, 1, 1, 1, 1] \]

Hamming distances:
\[ D = d_{ij} = \begin{pmatrix}
1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\
1 & 0 & 1 & 2 & 3 & 4 & 5 & 0 & 0 & 0 \\
2 & 1 & 0 & 1 & 2 & 3 & 4 & 5 & 0 & 0 \\
3 & 1 & 2 & 0 & 1 & 2 & 3 & 4 & 5 & 0 \\
4 & 1 & 2 & 3 & 0 & 1 & 2 & 3 & 4 & 5 \\
5 & 1 & 2 & 3 & 4 & 0 & 1 & 2 & 3 & 4 \\
6 & 1 & 2 & 3 & 4 & 5 & 0 & 1 & 2 & 3 \\
7 & 1 & 2 & 3 & 4 & 5 & 6 & 0 & 1 & 2 \\
8 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 0 & 1 \\
9 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 0 \\
10 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\
\end{pmatrix} \]

SUBJECT TO
\[ x_i + x_j - 1 \leq y_{ij} \]

Constraints

\[ \begin{align*}
X_1 + X_2 &= 1 \\
X_2 + X_3 &= 1 \\
X_3 + X_4 &= 1 \\
X_4 + X_5 &= 1 \\
X_5 + X_6 &= 1 \\
X_6 + X_7 &= 1 \\
X_7 + X_8 &= 1 \\
X_8 + X_9 &= 1 \\
X_9 + X_{10} &= 1
\end{align*} \]

\[ \begin{align*}
X_1 + X_2 &= 1 \\
X_2 + X_3 &= 1 \\
X_3 + X_4 &= 1 \\
X_4 + X_5 &= 1 \\
X_5 + X_6 &= 1 \\
X_6 + X_7 &= 1 \\
X_7 + X_8 &= 1 \\
X_8 + X_9 &= 1 \\
X_9 + X_{10} &= 1
\end{align*} \]

\[ \begin{align*}
X_1 + X_3 &= 1 \\
X_2 + X_4 &= 1 \\
X_3 + X_5 &= 1 \\
X_4 + X_6 &= 1 \\
X_5 + X_7 &= 1 \\
X_6 + X_8 &= 1 \\
X_7 + X_9 &= 1 \\
X_8 + X_{10} &= 1
\end{align*} \]

\[ \begin{align*}
X_1 + X_2 &= 1 \\
X_2 + X_3 &= 1 \\
X_3 + X_4 &= 1 \\
X_4 + X_5 &= 1 \\
X_5 + X_6 &= 1 \\
X_6 + X_7 &= 1 \\
X_7 + X_8 &= 1 \\
X_8 + X_9 &= 1 \\
X_9 + X_{10} &= 1
\end{align*} \]

\[ \begin{align*}
X_1 + X_2 &= 1 \\
X_2 + X_3 &= 1 \\
X_3 + X_4 &= 1 \\
X_4 + X_5 &= 1 \\
X_5 + X_6 &= 1 \\
X_6 + X_7 &= 1 \\
X_7 + X_8 &= 1 \\
X_8 + X_9 &= 1 \\
X_9 + X_{10} &= 1
\end{align*} \]

The solution \( x_1 = x_4 = x_7 = x_9 = 1 \),
\[ \text{i.e., process plans 1, 4, 7, 9 with the cost of 29.8 is obtained} \]

Tools selected: \( t_1, t_3, t_4 \)
Fixtures selected: \( t_1 \)