9.78

9.78 A 0.30-m-diameter cork ball ($SG = 0.21$) is tied to an object on the bottom of a river as is shown in Fig. P9.78. Estimate the speed of the river current. Neglect the weight of the cable and the drag on it.

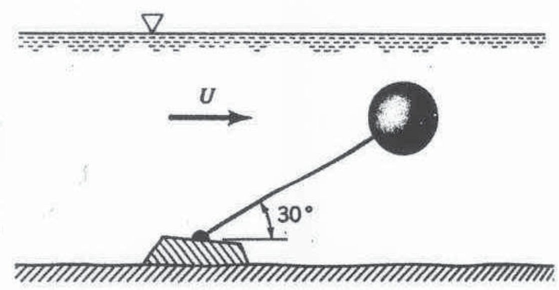
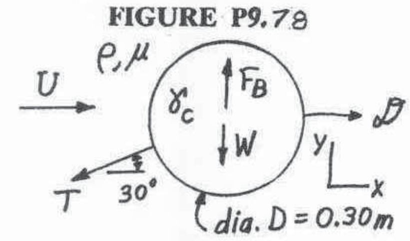


FIGURE P9.78



For the ball to remain stationary

$\Sigma F_x = 0$ and $\Sigma F_y = 0$

Thus, $\mathcal{D} = T\cos 30°$ or $T = \dfrac{\mathcal{D}}{\cos 30°}$

and

$F_B = W + T\sin 30°$

Hence, $F_B = W + \mathcal{D}\tan 30°$, where $F_B = \rho g \forall = \left(9.80\,\frac{kN}{m^3}\right)\left(\frac{4\pi}{3}\left(\frac{0.30}{2}m\right)^3\right)$

$= 0.1385\ kN$

and

$W = \gamma_c \forall = \left(\frac{\gamma_c}{\gamma}\right)\gamma\forall = (SG)F_B$

$= 0.21(0.1385\ kN)$

$= 0.0291\ kN$

Thus,

$0.1385\ kN = 0.0291\ kN + \mathcal{D}\tan 30°$

or

$\mathcal{D} = 0.189\ kN$, where $\mathcal{D} = C_D \frac{1}{2}\rho U^2 A = C_D U^2\left(\frac{1}{2}\right)\left(999\,\frac{kg}{m^3}\right)\left(\frac{\pi}{4}(0.3m)^2\right)$

$= 35.3\ C_D U^2\ N$, where $U \sim \frac{m}{s}$

Hence

$35.3 C_D U^2 = 189$ or $C_D U^2 = 5.35$ (1)

Also, $Re = \dfrac{UD}{\nu} = \dfrac{(0.3m)\,U}{1.12\times10^{-6}\,\frac{m^2}{s}} = 2.68\times10^5\,U$ (2)

and

from Fig. 9.21 (3)

Trial and error solution for U: Assume C_D; calculate U from Eq.(1) and Re from Eq.(2); check C_D from Eq.(3), the graph.

Assume $C_D = 0.5 \rightarrow U = 3.27\,\frac{m}{s} \rightarrow Re = 8.76\times10^5 \rightarrow C_D = 0.15 \neq 0.5$

Assume $C_D = 0.15 \rightarrow U = 5.97\,\frac{m}{s} \rightarrow Re = 1.60\times10^6 \rightarrow C_D = 0.20 \neq 0.15$

Assume $C_D = 0.19 \rightarrow U = 5.31\,\frac{m}{s} \rightarrow Re = 1.42\times10^6 \rightarrow C_D = 0.19$ (checks)

Thus, $\underline{\underline{U = 5.31\,\frac{m}{s}}}$