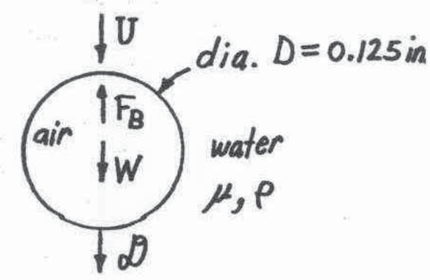
9.74

9.74 Compare the rise velocity of an $\frac{1}{8}$-in.-diameter air bubble in water to the fall velocity of an $\frac{1}{8}$-in.-diameter water drop in air. Assume each to behave as a solid sphere.



(a) air bubble in water: For steady rise $\sum F_z = 0$

or

$F_B = W + \mathscr{D}$, where $\mathscr{D} = \text{drag} = C_D \frac{1}{2}\rho U^2 \frac{\pi}{4} D^2$

$W = \text{weight} = \gamma_{air} \forall = \gamma_{air} \frac{4\pi}{3}\left(\frac{D}{2}\right)^3$

and

$F_B = \text{buoyant force} = \gamma_{H_2O} \forall = \gamma_{H_2O} \frac{4\pi}{3}\left(\frac{D}{2}\right)^3$

However, since $\gamma_{air} \ll \gamma_{H_2O}$ it follows that $W \ll F_B$

or

$F_B = \mathscr{D}$

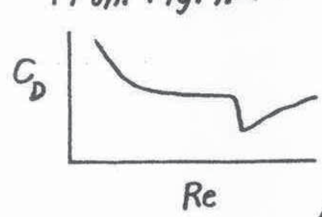
Hence, $g\rho \frac{4\pi}{3}\left(\frac{D}{2}\right)^3 = C_D \frac{1}{2}\rho U^2 \frac{\pi}{4} D^2$ or $U = \sqrt{\frac{4Dg}{3C_D}} = \sqrt{\frac{4\left(\frac{0.125}{12}\right)\text{ft}\left(32.2\frac{\text{ft}}{\text{s}^2}\right)}{3C_D}}$

or $U = \frac{0.669}{\sqrt{C_D}} \frac{\text{ft}}{\text{s}}$, where $C_D = C_D(Re)$ and $Re = \frac{UD}{\nu}$ (1)

or

$Re = \frac{\left(\frac{0.125}{12}\text{ft}\right)U}{1.21 \times 10^{-5}\frac{\text{ft}^2}{\text{s}}} = 861\,U$ (2)

From Fig. 9.21:



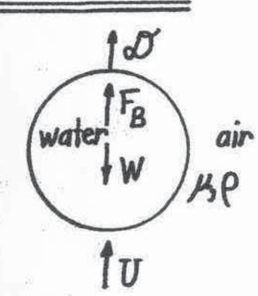
Trial and error solution for U: Assume C_D; obtain U from Eq.(1), Re from Eq.(2); check C_D from Eq.(3), the graph. (3)

Assume $C_D = 1 \rightarrow U = 0.669\frac{\text{ft}}{\text{s}} \rightarrow Re = 576 \rightarrow C_D = 0.5 \neq 1$

Assume $C_D = 0.5 \rightarrow U = 0.946\frac{\text{ft}}{\text{s}} \rightarrow Re = 815 \rightarrow C_D = 0.5$ (checks)

Thus, $U = 0.946\frac{\text{ft}}{\text{s}}$

(b) water drop in air: Since $\gamma_{air} \ll \gamma_{H_2O}$, $F_B \ll W$



Thus, $W = \mathscr{D}$, or $\gamma_{H_2O} \frac{4\pi}{3}\left(\frac{D}{2}\right)^3 = C_D \frac{1}{2}\rho U^2 \frac{\pi}{4} D^2$

or $U = \sqrt{\frac{4D\gamma_{H_2O}}{3\rho C_D}} = \left[\frac{4\left(\frac{0.125}{12}\text{ft}\right)\left(62.4\frac{\text{lb}}{\text{ft}^3}\right)}{3\left(2.38 \times 10^{-3}\frac{\text{slugs}}{\text{ft}^3}\right)C_D}\right]^{1/2} = \frac{19.1}{\sqrt{C_D}} \frac{\text{ft}}{\text{s}}$ (4)

Also,

$Re = \frac{UD}{\nu} = \frac{\left(\frac{0.125}{12}\text{ft}\right)U}{1.57 \times 10^{-4}\frac{\text{ft}^2}{\text{s}}} = 66.3\,U$ (5)

Trial and error solution of Eqs. (4), (5), and graph (3):

Assume $C_D = 0.5 \rightarrow U = 27.0\frac{\text{ft}}{\text{s}} \rightarrow Re = 1790 \rightarrow C_D = 0.4 \neq 0.5$

Assume $C_D = 0.4 \rightarrow U = 30.2\frac{\text{ft}}{\text{s}} \rightarrow Re = 2000 \rightarrow C_D = 0.4$ (checks)

Thus, $U = 30.2\frac{\text{ft}}{\text{s}}$

Note: Because of the graph (Fig. 9.21) the answers are not accurate to three significant figures.