

8.102

8.102. Determine the diameter of a steel pipe that is to carry 2,000 gal/min of gasoline with a pressure drop of 5 psi per 100 ft of horizontal pipe.

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + f \frac{L}{D} \frac{V^2}{2g}, \text{ where } z_1 = z_2 \text{ and } V_1 = V_2$$

Thus,

$$p_1 - p_2 = f \frac{L}{D} \frac{1}{2} \rho V^2 \text{ with } p_1 - p_2 = 5 \frac{\text{lb}}{\text{in}^2}, (144 \frac{\text{in}^2}{\text{ft}^2}), L = 100 \text{ ft}, \quad (1)$$

$$V = \frac{Q}{A} = \frac{(2000 \frac{\text{gal}}{\text{min}})(\frac{1 \text{ min}}{60 \text{ s}})(231 \frac{\text{in}^3}{\text{gal}})(\frac{1}{1728 \frac{\text{in}^3}{\text{ft}^3}})}{\frac{\pi}{4} D^2}, \text{ or } V = \frac{5.67}{D^2} \frac{\text{ft}}{\text{s}} \text{ with } D \sim \text{ft}$$

Hence, Eq. (1) gives:

$$5(144) \frac{\text{lb}}{\text{ft}^2} = f \left(\frac{100 \text{ ft}}{D \text{ ft}} \right) \frac{1}{2} (1.32 \frac{\text{slug}}{\text{ft}^3}) \left(\frac{5.67}{D^2} \frac{\text{ft}}{\text{s}} \right)^2$$

or

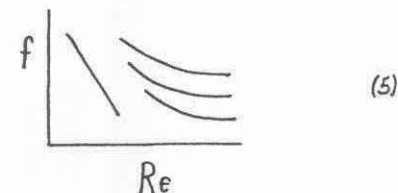
$$D = 1.24 f^{1/5} \quad (2)$$

$$\text{Also, } Re = \frac{\rho V D}{\mu} = \frac{(1.32 \frac{\text{slug}}{\text{ft}^3}) (\frac{5.67}{D^2} \frac{\text{ft}}{\text{s}}) D \text{ ft}}{6.5 \times 10^{-6} \frac{\text{lb} \cdot \text{s}}{\text{ft}^2}}, \text{ or } Re = \frac{1.15 \times 10^6}{D} \quad (3)$$

and

$$\frac{\epsilon}{D} = \frac{0.00015}{D}, \text{ where } D \sim \text{ft} \quad (4)$$

Finally, the fourth equation is the Moody chart (or the Colebrook equation)



Note: 4 equations (2), (3), (4), and (5) and 4 unknowns (f , $\frac{\epsilon}{D}$, D , Re)

Trial and error solution:

$$\begin{aligned} \text{Guess } f = 0.02 &\xrightarrow{(2)} D = 0.567 \text{ ft} \begin{cases} (3) \rightarrow Re = 2.03 \times 10^6 \\ (4) \rightarrow \frac{\epsilon}{D} = 0.000265 \end{cases} \xrightarrow{(5)} f = 0.0148 \neq 0.02 \\ \text{Thus, the guessed value is not correct.} \\ \text{Guess } f = 0.0148 &\xrightarrow{(2)} D = 0.534 \text{ ft} \begin{cases} (3) \rightarrow Re = 2.15 \times 10^6 \\ (4) \rightarrow \frac{\epsilon}{D} = 0.000281 \end{cases} \xrightarrow{(5)} f = 0.0150 \approx 0.0148 \end{aligned}$$

$$\text{Thus, } D = 1.24 (0.0150)^{1/5} = \underline{0.535 \text{ ft}}$$

By using the Colebrook equation, Eq. 8.35, rather than the Moody chart, Eq. (5), we have

$$\frac{1}{\sqrt{f}} = -2.0 \log \left[\frac{\epsilon/D}{3.7} + \frac{2.51}{Re \sqrt{f}} \right] \text{ which, using Eqs (2), (3), and (4) is,}$$

$$\frac{1}{(D/1.24)^{5/2}} = -2.0 \log \left[\frac{0.00015}{3.7 D} + \frac{2.51 D}{1.15 \times 10^6 (D/1.24)^{5/2}} \right]$$

Using a computer root-finding program to solve Eq. (6) gives $D = 0.536 \text{ ft}$ which is consistent with the above trial and error solution.