7.22 The pressure drop, Δp , along a straight pipe of diameter D has been experimentally studied, and it is observed that for laminar flow of a given fluid and pipe, the pressure drop varies directly with the distance, ℓ , between pressure taps. Assume that Δp is a function of D and ℓ , the velocity, V, and the fluid viscosity, μ . Use dimensional analysis to deduce how the pressure drop varies with pipe diameter.

$$\Delta p = f(D, l, V, \mu)$$

Ap = FL-2 D= L l= L V= LT-1 p= FL-2T

From the pi theorem, 5-3=2 pi terms required.

By inspection, for TT, (containing Ap):

$$TT_{I} = \frac{\Delta \rho D}{\mu V} \doteq \frac{(FL^{-2})(L)}{(FL^{-2}T)(LT^{-1})} \doteq F^{\circ}L^{\circ}T^{\circ}$$

Check using MLT:

$$\frac{ApD}{\mu\nu} = \frac{(ML^{-1}T^{-2})(L)}{(ML^{-1}T^{-1})(LT^{-1})} = M^{\circ}L^{\circ}T^{\circ} :: oK$$

For TT2 (containing &):

$$\pi_2 = \frac{\ell}{D}$$

Which is obviously dimensionless. Thus,

$$\frac{\Delta p \, D}{\mu \, V} = \phi \left(\frac{l}{D}\right) \tag{1}$$

From the statement of the problem, $\Delta p \propto l$ so that Eq. (1) must be of the form

$$\frac{ApD}{\mu\nu} = K \frac{l}{D}$$

Where K is some constant. It Thus follows that

for a given velocity.