6.18

6.18 The velocity components of an incompressible, two-dimensional velocity field are given by the equations

$$u = 2xy$$
$$v = x^2 - y^2$$

Show that the flow is irrotational and satisfies conservation of mass.

If the two-dimensional flow is irrotational,

$$\omega_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) = 0$$

For the velocity distribution given,

$$\frac{\partial v}{\partial x} = 2x \qquad \frac{\partial u}{\partial y} = 2x$$

Thus,

$$\omega_z = \frac{1}{2}(2x - 2x) = 0$$

and the flow is irrotational.

To satisfy conservation of mass,

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

Since,

$$\frac{\partial u}{\partial x} = 2y \qquad \frac{\partial v}{\partial y} = -2y$$

then

$$2y - 2y = 0$$

and conservation of mass is satisfied.