6.17

**6.17** **The velocity components in an incompressible, two-dimensional flow field are given by the equations**

$$u = x^2$$

$$v = -2xy + x$$

**Determine, if possible, the corresponding stream function.**

From the definition of the stream function,

$$u = \frac{\partial \psi}{\partial y} \qquad v = -\frac{\partial \psi}{\partial x} \qquad \text{(Eqs. 6.37)}$$

so that for the velocity distribution given,

$$\frac{\partial \psi}{\partial y} = x^2 \qquad (1)$$

$$\frac{\partial \psi}{\partial x} = 2xy - x \qquad (2)$$

Integrate Eq.(1) with respect to $y$ to obtain

$$\int d\psi = \int x^2 dy + f_1(x)$$

or

$$\psi = x^2 y + f_1(x) \qquad (3)$$

Similarly, integrate Eq.(2) with respect to $x$ to obtain

$$\int d\psi = \int (2xy - x)\, dx + f_2(y)$$

or

$$\psi = x^2 y - \frac{x^2}{2} + f_2(y) \qquad (4)$$

Thus, to satisfy both Eqs.(3) and (4)

$$\psi = x^2 y - \frac{x^2}{2} + C$$

where $C$ is an arbitrary constant.