4.31

4.31 As a valve is opened, water flows through the diffuser shown in Fig. P4.31 at an increasing flowrate so that the velocity along the centerline is given by $\mathbf{V} = u\hat{\mathbf{i}} = V_0(1 - e^{-ct})(1 - x/\ell)\hat{\mathbf{i}}$, where u_0, c, and ℓ are constants. Determine the acceleration as a function of x and t. If $V_0 = 10$ ft/s and $\ell = 5$ ft, what value of c (other than $c = 0$) is needed to make the acceleration zero for any x at $t = 1$ s? Explain how the acceleration can be zero if the flowrate is increasing with time.

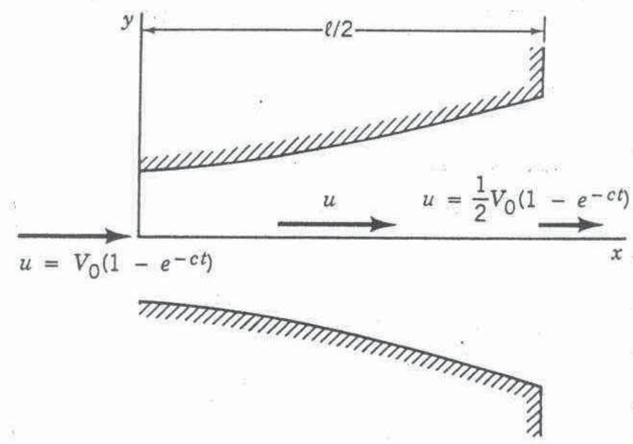

FIGURE P4.31

$\vec{a} = \frac{\partial \vec{V}}{\partial t} + \vec{V}\cdot\nabla\vec{V}$ With $u = u(x,t)$, $v = 0$, and $w = 0$ this becomes

$\vec{a} = \left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x}\right)\hat{i} = a_x\hat{i}$, where $u = V_0(1-e^{-ct})(1-\frac{x}{\ell})$

Thus,

$$a_x = V_0\left(1-\frac{x}{\ell}\right)c\,e^{-ct} + V_0^2(1-e^{-ct})^2\left(1-\frac{x}{\ell}\right)\left(-\frac{1}{\ell}\right)$$

or

$$a_x = V_0\left(1-\frac{x}{\ell}\right)\left[c\,e^{-ct} - \frac{V_0}{\ell}(1-e^{-ct})^2\right]$$

If $a_x = 0$ for any x at $t = 1$ s we must have

$\left[c\,e^{-ct} - \frac{V_0}{\ell}(1-e^{-ct})^2\right] = 0$ With $V_0 = 10$ and $\ell = 5$

$c\,e^{-c} - \frac{10}{5}(1-e^{-c})^2 = 0$ The solution (root) of this equation is $\underline{c = 0.490\ \frac{1}{s}}$

For the above conditions the local acceleration ($\frac{\partial u}{\partial t} > 0$) is precisely balanced by the convective deceleration ($u\frac{\partial u}{\partial x} < 0$). The flowrate increases with time, but the fluid flows to an area of lower velocity.