4.10

4.10 The velocity field of a flow is given by $u = -V_0 y/(x^2 + y^2)^{1/2}$ and $v = V_0 x/(x^2 + y^2)^{1/2}$, where V_0 is a constant. Where in the flow field is the speed equal to V_0? Determine equation of the streamlines and discuss the various characteristics of this flow.

$u = -V_0 \frac{y}{(x^2+y^2)^{1/2}}$, $v = V_0 \frac{x}{(x^2+y^2)^{1/2}}$ so that

$V = \sqrt{u^2 + v^2} = \left[\frac{V_0^2 (y^2+x^2)}{(x^2+y^2)}\right]^{1/2} = V_0$

Thus, $V = V_0$ throughout the entire flow field

Streamlines are given by

$\frac{dy}{dx} = \frac{v}{u} = \frac{x}{-y}$ or $-y\,dy = x\,dx$ which can be integrated to give $x^2+y^2 = \text{const.}$

Thus, the fluid flow with circular streamlines and the speed is constant throughout.