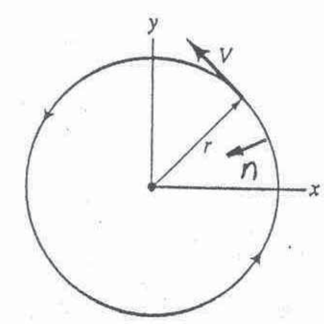
3.14

3.14 Water in a container and air in a tornado flow in horizontal circular streamlines of radius r and speed V as shown in Video V3.6 and Fig. P3.14. Determine the radial pressure gradient, $\partial p/\partial r$, needed for the following situations: (a) The fluid is water with r = 3 in. and V = 0.8 ft/s. (b) The fluid is air with r = 300 ft and V = 200 mph.



■ FIGURE P3.14

For curved streamlines,

$$-\frac{dp}{dn} = \frac{\rho V^2}{\mathcal{R}} + \gamma \frac{dz}{dn}, \text{ or with } \frac{dz}{dn} = 0 \text{ (horizontal streamlines), } \mathcal{R} = r,$$

and $\frac{d}{dn} = -\frac{d}{dr}$ this becomes

$$\frac{dp}{dr} = \frac{\rho V^2}{r}$$

a) With $r = \frac{3}{12}$ ft and $V = 0.8 \frac{ft}{s}$ and water ($\rho = 1.94 \frac{slugs}{ft^3}$),

$$\frac{dp}{dr} = \frac{1.94 \frac{slugs}{ft^3}\left(0.8 \frac{ft}{s}\right)^2}{\left(\frac{3}{12} ft\right)} = 4.97 \frac{slugs}{ft^2 \cdot s^2} = \underline{\underline{4.97 \frac{lb}{ft^3}}}$$

(b) With $r = 300 ft$ and $V = 200 \, mph \left(\frac{88 \frac{ft}{s}}{60 \, mph}\right) = 293 \frac{ft}{s}$

and air ($\rho = 0.00238 \frac{slugs}{ft^3}$),

$$\frac{dp}{dr} = \frac{0.00238 \frac{slugs}{ft^3}\left(293 \frac{ft}{s}\right)^2}{300 ft} = 0.681 \frac{slugs}{ft^2 \cdot s^2} = \underline{\underline{0.681 \frac{lb}{ft^3}}}$$