2.23

2.23 As shown in Fig. 2.6 for the U.S. standard atmosphere, the troposphere extends to an altitude of 11 km where the pressure is 22.6 kPa (abs). In the next layer, called the stratosphere, the temperature remains constant at $-56.5\ ^\circ C$. Determine the pressure and density in this layer at an altitude of 15 km. Assume $g = 9.77\ m/s^2$ in your calculations. Compare your results with those given in Table C.2 in Appendix C.

For isothermal conditions,

$$p_2 = p_1 e^{\frac{-g(z_2 - z_1)}{RT_0}} \qquad \text{(Eq. 2.10)}$$

Let $z_1 = 11\ km$, $p_1 = 22.6\ kPa$, $R = 287\ \frac{J}{kg \cdot K}$, $g = 9.77\ \frac{m}{s^2}$, and $T_0 = -56.5^\circ C + 273.15 = 216.65\ K$.

Thus,

$$p_2 = (22.6\ kPa)\, e^{-\left[\frac{\left(9.77\ \frac{m}{s^2}\right)\left(15 \times 10^3\ m - 11 \times 10^3\ m\right)}{\left(287\ \frac{J}{kg \cdot K}\right)(216.65\ K)}\right]}$$

$$= \underline{\underline{12.1\ kPa}}$$

Also,

$$\rho_2 = \frac{p}{RT} = \frac{12.1 \times 10^3\ \frac{N}{m^2}}{\left(287\ \frac{J}{kg \cdot K}\right)(216.65\ K)} = \underline{\underline{0.195\ \frac{kg}{m^3}}}$$

(From Table C.2 in Appendix C, $p_2 = 12.11\ kPa$ and $\rho_2 = 0.1948\ \frac{kg}{m^3}$.)