CHAPTER 2 BACKGROUND THEORY

2.1 Overview of Maneuvering Simulations

The Maneuvering Committee (MC) of the 24th International Towing Tank Conference (ITTC) reviewed state-of-the-art progress in maneuvering predictions, and categorized typical maneuvering prediction methods into three groups: No Simulation, System Based Simulation, and CFD Based Simulation methods.

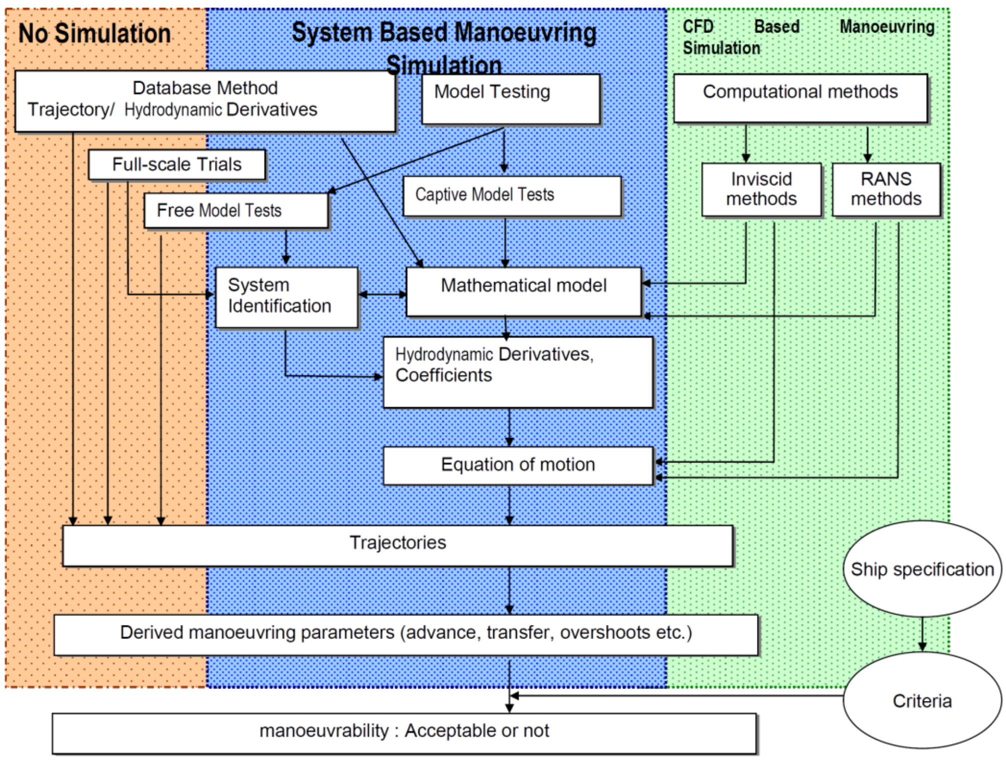

Figure 2-1 Overview of Maneuvering prediction methods (Proceedings of 25th ITTC, Vol. I, pp. 145).

The No Simulation method needs no mathematical model and thus no hydrodynamic derivative or maneuvering coefficient. Maneuvering parameters such as ship advance, transfer, overshoot, and etc. are directly measured from the full-scale trial or model-scale free-model test by measuring the ship trajectories or by using a database of existing full- and/or model-scale data.

The System Based Simulation method, by contrast, simulates the ship trajectories by solving the motion equations using appropriate mathematical modeling along with hydrodynamic derivatives (or maneuvering coefficients). This method includes 1) database, 2) model testing, and 3) system identification methods. First, the database method establishes an empirical formula or regression equations from databases of full- and/or model-scale test results to obtain hydrodynamic derivatives (Oltmann 1992, Wagner-Smitt 1971, Norrbin 1971, Inoue et al. 1981, Clarke et al. 1983, Kijima et al. 1990 and 1993). The database can be also combined with theoretical models such as the Japanese Mathematical Model Group (MMG) model (Kijima et al. 1993) or the cross-flow drag model (Hooft, 1994). These methods are simple and quick to use, but the prediction accuracy and/or reliability can be limited when the ship dimensions are outside the database. Next, the model test method includes free- and captive-model tests. For free model tests (Martinussen et al. 1987), a self-propelled scale model ship is remotely controlled performing definitive maneuvers such as turning circle, zig-zag, and reverse spiral to evaluate turning performance and course keeping stability. This method is direct and effective since the maneuvering parameters are directly obtained without simulation, but with issues about viscous scale effects (Burcher 1975). On the other hand, the captive model tests are based on mathematical modeling of motion equations. For the tests, a model-scale ship is forced to move in prescribed motions over a range of parameters such as drift angle, sway/yaw motion amplitude and frequency, rudder angle, etc. to obtain the relevant hydrodynamic derivatives. Details of the captive model tests are provided in the following Section 2.3. Lastly, the system identification method (Artyszuk 2003, Hess and Faller 2000, Moreira and Soares 2003, Oltmann 2003, Viviani et al. 2003, Depascale et al. 2002, Yoon et al. 2003) obtains hydrodynamic derivatives from full-scale sea trial or free-model test results using measured ship motion and rudder angle as input parameters.

CFD Based Simulation method also simulates the ship trajectory to predict the maneuvering parameters similarly as the System Based Simulation method but by using numerical schemes to evaluated the hydrodynamic derivatives of the mathematic models used or to solve the motion equations directly.

2.2 Mathematic Modeling and Hydrodynamic Derivatives

The generalized motion equations for a rigid vessel in a ship-fixed, non-inertial frame of reference xyz that is moving relative to an Earth-fixed, inertial reference frame $x_E y_E z_E$ (Fig. 2-2) can be derived as (Fossen 1994):

$$m[\dot{u} - rv + wq - x_G(q^2 + r^2) + y_G(pq - \dot{r}) + z_G(pr + \dot{q})] = X \tag{2.1a}$$

$$m[\dot{v} - wp + ur - y_G(r^2 + p^2) + z_G(qr - \dot{p}) + x_G(qp + \dot{r})] = Y \tag{2.1b}$$

$$m[\dot{w} - uq + vp - z_G(p^2 + q^2) + x_G(rp - \dot{q}) + y_G(rq + \dot{p})] = Z \tag{2.1c}$$

$$I_x\dot{p} + (I_z - I_y)qr + m[y_G(\dot{w} - uq + vp) - z_G(\dot{v} - wp + ur)] = K \tag{2.1d}$$

$$I_y\dot{q} + (I_x - I_z)rp + m[z_G(\dot{u} - vr + wq) - x_G(\dot{w} - uq + vp)] = M \tag{2.1e}$$

$$I_z\dot{r} + (I_y - I_x)pq + m[x_G(\dot{v} - wp + ur) - y_G(\dot{u} - vr + wq)] = N \tag{2.1f}$$

The origin of the ship-fixed reference frame is located at the mid-ship position. The x, y, and z axes correspond to the longitudinal, lateral, and vertical direction of the vessel, respectively, so that the products of moment of inertia such as I_{xy}, I_{xz}, or I_{yz} vanish from the motion equations. In the equations, X, Y, Z are the external forces acting on the vessel in surge, x, sway, y, and heave, z directions, respectively. K, M, N are the external angular moments in roll, ϕ, pitch, θ, and yaw, ψ, directions, respectively. m is the mass of the vessel and I_x, I_y, I_z are the moments of inertia of the vessel with respect to each axis. x_G, y_G, z_G are the location of the center of gravity of the vessel. u, v, w are surge, sway, and heave velocities, $\dot{x}$, $\dot{y}$, $\dot{z}$, respectively, and $\dot{u}$, $\dot{v}$, $\dot{r}$ are surge, sway, and heave accelerations, $\ddot{x}$, $\ddot{y}$, $\ddot{z}$, respectively. p, q, r are roll, pitch, yaw rates, $\dot{\phi}$, $\dot{\theta}$, $\dot{\psi}$, respectively, and $\dot{p}$, $\dot{q}$, $\dot{r}$ are roll, pitch, yaw accelerations, $\ddot{\phi}$, $\ddot{\theta}$, $\ddot{\psi}$, respectively.

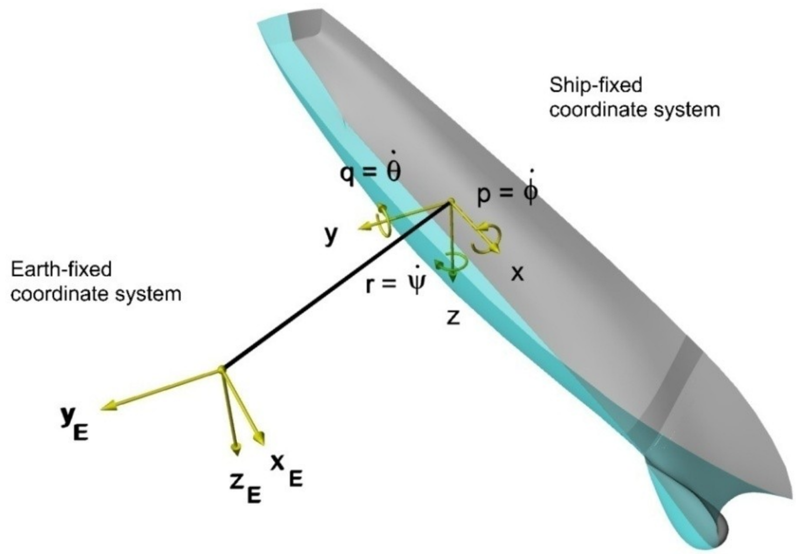

Figure 2-2 Earth- and ship-fixed coordinate systems.

For maneuvering applications of the equations (2.1) for surface ships moving on unbounded, calm, and deep water, it is typically assumed that the heave, roll, and pitch motions can be neglected such that $w = p = q = \dot{w} = \dot{p} = \dot{q} = 0$ and that the vessel geometry has the xz-plane symmetry, i.e. $y_G = 0$. Then, the equations reduce to the following equations:

$$m(\dot{u} - rv - x_G r^2) = X \quad (2.2a)$$

$$m(\dot{v} + ur - x_G \dot{r}) = Y \quad (2.2b)$$

$$I_z \dot{r} + m x_G (\dot{v} + ur) = N \quad (2.2c)$$

for surge, sway, and yaw, respectively. In general the external forces and moment X, Y, N at the right hand sides of the equations (2.2) include hydrodynamic forces due to the surrounding fluid, control surface forces such as rudder forces, and propulsion forces such as propeller forces, which need to be described in proper mathematical forms for the motion equations to be solved. One of the common mathematic modeling of those forces is by assuming that the forces are functions of ship motion parameters u, v, r, $\dot{u}$, $\dot{v}$, $\dot{r}$ and rudder deflection angle δ (Abkowitz, 1964) based on the 'quasi-steady state' assumption which states that the value of the forces at any instant depends on the motion parameters defining the instantaneous motion of the vessel.

$$\left.\begin{matrix} X \\ Y \\ N \end{matrix}\right\} = f(u, v, r, \dot{u}, \dot{v}, \dot{r}, \delta) \qquad (2.3)$$

Abkowitz (1964) also proposed to use a 3rd-order Taylor Series expansion of the equation (2.3) with following additional assumptions:

a) Forces and moments have appropriate port and starboard symmetry except for a constant force and moment caused by the propeller, and
b) There are no second- or higher-order acceleration terms, and that cross-coupling between acceleration and velocity parameters is negligible,

as per re-stated by Strom-Tejsen and Chislett (1966). Then, for small disturbances of the ship motions from a reference state, i.e. steady straight advancing with a constant speed U, the equation (2.3) are written as following (Strom-Tejsen and Chislett 1966):

$$\begin{aligned} X = {} & X_* + X_{\dot{u}}\dot{u} + X_u \Delta u + X_{uu} \Delta u^2 + X_{uuu} \Delta u^3 + \\ & X_{vv} v^2 + X_{rr} r^2 + X_{\delta\delta} \delta^2 + X_{vvu} v^2 \Delta u + X_{rru} r^2 \Delta u + X_{\delta\delta u} \delta^2 \Delta u + \\ & X_{vr} vr + X_{v\delta} v\delta + X_{r\delta} r\delta + X_{vru} vr\Delta u + X_{v\delta u} v\delta\Delta u + X_{r\delta u} r\delta\Delta u \qquad (2.4a) \end{aligned}$$

$$\begin{aligned} Y = {} & Y_* + Y_u \Delta u + Y_{uu} \Delta u^2 + Y_{uuu} \Delta u^3 + \\ & Y_{\dot{v}}\dot{v} + Y_v v + Y_{vvv} v^3 + Y_{vrr} vr^2 + Y_{v\delta\delta} v\delta^2 + Y_{vu} v\Delta u + Y_{vuu} v\Delta u^2 + \\ & Y_{\dot{r}}\dot{r} + Y_r r + Y_{rrr} r^3 + Y_{rvv} rv^2 + Y_{r\delta\delta} r\delta^2 + Y_{ru} r\Delta u + Y_{ruu} r\Delta u^2 + \end{aligned}$$

$$Y_\delta \delta + Y_{\delta\delta\delta} \delta^3 + Y_{\delta vv} \delta v^2 + Y_{\delta rr} \delta r^2 + Y_{\delta u} \delta \Delta u + Y_{\delta uu} \delta \Delta u^2 +$$

$$Y_{\delta\delta\delta u} \delta^3 \Delta u + Y_{vr\delta} vr\delta \quad (2.4b)$$

$$N = N_* + N_u \Delta u + N_{uu} \Delta u^2 + N_{uuu} \Delta u^3 +$$

$$N_{\dot{v}} \dot{v} + N_v v + N_{vvv} v^3 + N_{vrr} vr^2 + N_{v\delta\delta} v\delta^2 + N_{vu} v\Delta u + N_{vuu} v\Delta u^2 +$$

$$N_{\dot{r}} \dot{r} + N_r r + N_{rrr} r^3 + N_{rvv} rv^2 + N_{r\delta\delta} r\delta^2 + N_{ru} r\Delta u + N_{ruu} r\Delta u^2 +$$

$$N_\delta \delta + N_{\delta\delta\delta} \delta^3 + N_{\delta vv} \delta v^2 + N_{\delta rr} \delta r^2 + N_{\delta u} \delta \Delta u + N_{\delta uu} \delta \Delta u^2 +$$

$$N_{\delta\delta\delta u} \delta^3 \Delta u + N_{vr\delta} vr\delta \quad (2.4c)$$

where $\Delta u \equiv u - U$ is the disturbance in surge velocity. The terms X_*, Y_*, N_* are the reference steady state values of X, Y, N, respectively. Typically, X_* is zero for ships advancing straight with a constant speed as the ship total resistance R_T is balanced by the propeller thrust T, however, Y_* and N_* may have non-zero values when the ship has a single propeller or multiple propellers rotating in the same direction. The coefficients of Taylor Series terms at the right hand sides of (2.4) with subscripts of motion parameters, such as $X_{\dot{u}} \equiv \partial X/\partial \dot{u}$ or $X_{vv} \equiv \frac{1}{2}\partial^2 X/\partial v^2$, are the reduced expressions of the Taylor Series expansion following the simplified derivative notation of SNAME (Nomenclature, 1952), so-called 'hydrodynamic derivatives' or 'maneuvering coefficients', evaluated at the reference steady state. Note that, although the Taylor Series were assumed as 3rd-order expansions, Strom-Tejsen and Chislett (1966) also used fourth-order as well for the rudder force terms such as $Y_{\delta\delta\delta u} \delta^3 \Delta u$ and $N_{\delta\delta\delta u} \delta^3 \Delta u$ to obtain sufficient flexibility in expressing the influence of surge velocity on the rudder action. Note also that the surge velocity expansion terms for Y and N such as $Y_u \Delta u$, $Y_{uu} \Delta u^2$, $Y_{uuu} \Delta u^3$ and $N_u \Delta u$, $N_{uu} \Delta u^2$, $N_{uuu} \Delta u^3$ in (2.4) replaced the terms $Y_{*u} \Delta u$, $Y_{*uu} \Delta u^2$ and $N_{*u} \Delta u$, $N_{*uu} \Delta u^2$, respectively, in Strom-Tejsen and Chislett (1966) as the former expressions are considered to be more consistent with the mathematical definitions of Taylor Series expansion in that the reference state values Y_* or N_* are not expanded.

2.3 PMM Tests

General descriptions and procedures of PMM tests including the static drift, pure sway, pure yaw, and yaw and drift tests and determination of hydrodynamic derivatives are provided. The procedures for rudder related tests such as static rudder, static drift and rudder, and yaw and rudder tests are not provided herein as the present research objective is focused on the PMM applications for a bare hull form, i.e. without rudders, propellers, and appendages except for bilge keels.

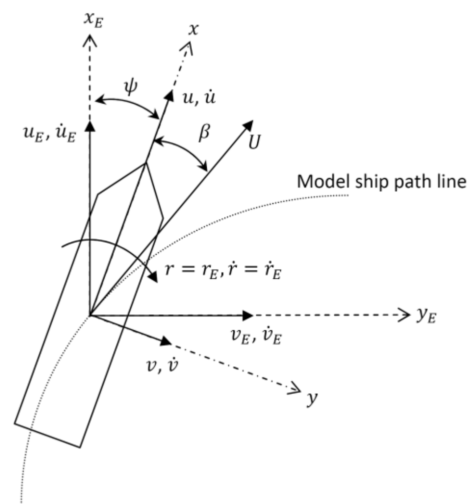

Figure 2-3 General PMM test coordinate system and motion parameters.

2.3.1 Definitions of Motions

Two coordinate systems are shown in Fig. 2-3: the Earth-fixed $x_E y_E$-coordinate system (dashed arrows) and the ship-fixed xy-coordinate system (dash-dot arrows). The Earth-fixed coordinates are fixed at the towing tank with x_E and y_E coordinates aligned with the longitudinal and lateral directions of towing tank, respectively. The ship-fixed coordinates are moving with the model with x and y coordinates aligned with the longitudinal and lateral directions of the model, respectively. For convenience, in the figure the Earth-fixed coordinate system is shown overlaid on the ship-fixed coordinate system at a certain instant. Vectors u_E and v_E are the velocities and $\dot{u}_E$ are $\dot{v}_E$ are the accelerations of the model in the x_E and y_E directions of the Earth-fixed coordinate system, respectively; and u and v are the velocities and $\dot{u}$ and $\dot{v}$ are the accelerations of model in the x and y directions of the ship-fixed coordinate system, respectively. The advance speed U is the resultant of u_E and v_E or the resultant of u and v such that

$$U = \sqrt{u_E^2 + v_E^2} = \sqrt{u^2 + v^2} \tag{2.5}$$

always tangent to the model path line (dotted line) that is the trajectory of the mid-ship point. Drift angle, β, is defined as the model orientation with respect to U, i.e., the actual direction of model with respect to its heading, which can be written as

$$\beta = -\arctan(v/u) \tag{2.6}$$

Heading (or yaw angle) ψ is defined as the model orientation with respect to a reference direction, x_E. Note that yaw rate $\dot{\psi}$ and acceleration $\ddot{\psi}$ are identical in both the Earth-fixed and the ship-fixed coordinate systems, i.e. $r_E = r = \dot{\psi}$ and $\dot{r}_E = \dot{r} = \ddot{\psi}$. Lastly, the vector transformations between the Earth- and ship-fixed coordinate systems are given as following:

$$u = u_E \cos\psi + v_E \sin\psi \tag{2.7a}$$

$$v = -u_E \sin\psi + v_E \cos\psi \tag{2.7b}$$

$$r = r_E \tag{2.7c}$$

$$\dot{u} = \dot{u}_E \cos\psi + \dot{v}_E \sin\psi + r_E(-u_E \sin\psi + v_E \cos\psi) \tag{2.7d}$$

$$\dot{v} = -\dot{u}_E \sin\psi + \dot{v}_E \cos\psi - r_E(u_E \cos\psi + v_E \sin\psi) \tag{2.7e}$$

$$\dot{r} = \dot{r}_E \tag{2.7f}$$

2.3.2 PMM Motions

PMM motions are the forced model trajectories comprised of three basic motions x_E, y_E, and ψ described in the $x_E y_E$-coordinate system:

$$x_E = U_C t \quad (2.8)$$

$$y_E = -y_{max} \sin \omega t \quad (2.9)$$

$$\psi = -\arctan(\varepsilon \cos \omega t) + \beta \quad (2.10)$$

where U_C is the towing speed, y_{max} is the sway amplitude, and ε is the maximum tangent of model trajectory defined as

$$\varepsilon = \left(\frac{dy_E}{dx_E}\right)_{max} = \left(\frac{dy_E/dt}{dx_E/dt}\right)_{max} = \frac{y_{max}\,\omega}{U_C} \quad (2.11)$$

The x_E in (2.8) corresponds to straight advancing motion with speed U_C along the towing tank longitudinal direction. The y_E in (2.9) is a sinusoidal lateral motion with an amplitude y_{max} and frequency ω. The ψ in (2.10) is a combination of a sinusoidal yaw motion and any drift angle β. For static drift test, $y_{max} = \varepsilon = \omega = 0$ in (2.9) and (2.10) and β is a fixed value in time, which corresponds to an oblique towing motion as shown in Fig. 2-4 (a) and (e). For pure sway test, y_{max} and ω are non-zero values in (2.9) thus a sinusoidal lateral motion but the model heading is kept in straight, i.e $\psi = 0$ in (2.10), as illustrated in Fig. 2-4 (b), which makes a continuously changing drift angle $\beta = \arctan(\varepsilon \cos \omega t)$ from (2.10) as shown in Fig. 2-4 (f). For pure yaw test, y_{max} and ω are non-zero in (2.9) and (2.10) similarly as pure sway test but $\beta = 0$ in (2.10), then the model is always tangent to its path-line as shown Fig. 2-4 (c) and (g). For yaw and drift test, y_{max} and ω are the same as for pure yaw test but β is set to a non-zero constant value in (2.10), which makes an asymmetric yaw motion as shown in Fig. 2-4 (d) and (h). For all tests, U_C in (2.8) is constant in time. From those model trajectories, the model velocities and accelerations in the Earth-fixed coordinates, i.e. $u_E = \dot{x}_E$, $v_E = \dot{y}_E$, $r_E = \dot{\psi}_E$, $\dot{u}_E = \ddot{x}_E$, $\dot{v}_E = \ddot{y}_E$, and $\dot{r}_E = \ddot{\psi}_E$, and in the ship-fixed coordinates u, v, r, $\dot{u}$, $\dot{v}$, and $\dot{r}$ as per the relationships (2.7) are summarized in Tables 2-1 and 2-2, respectively.

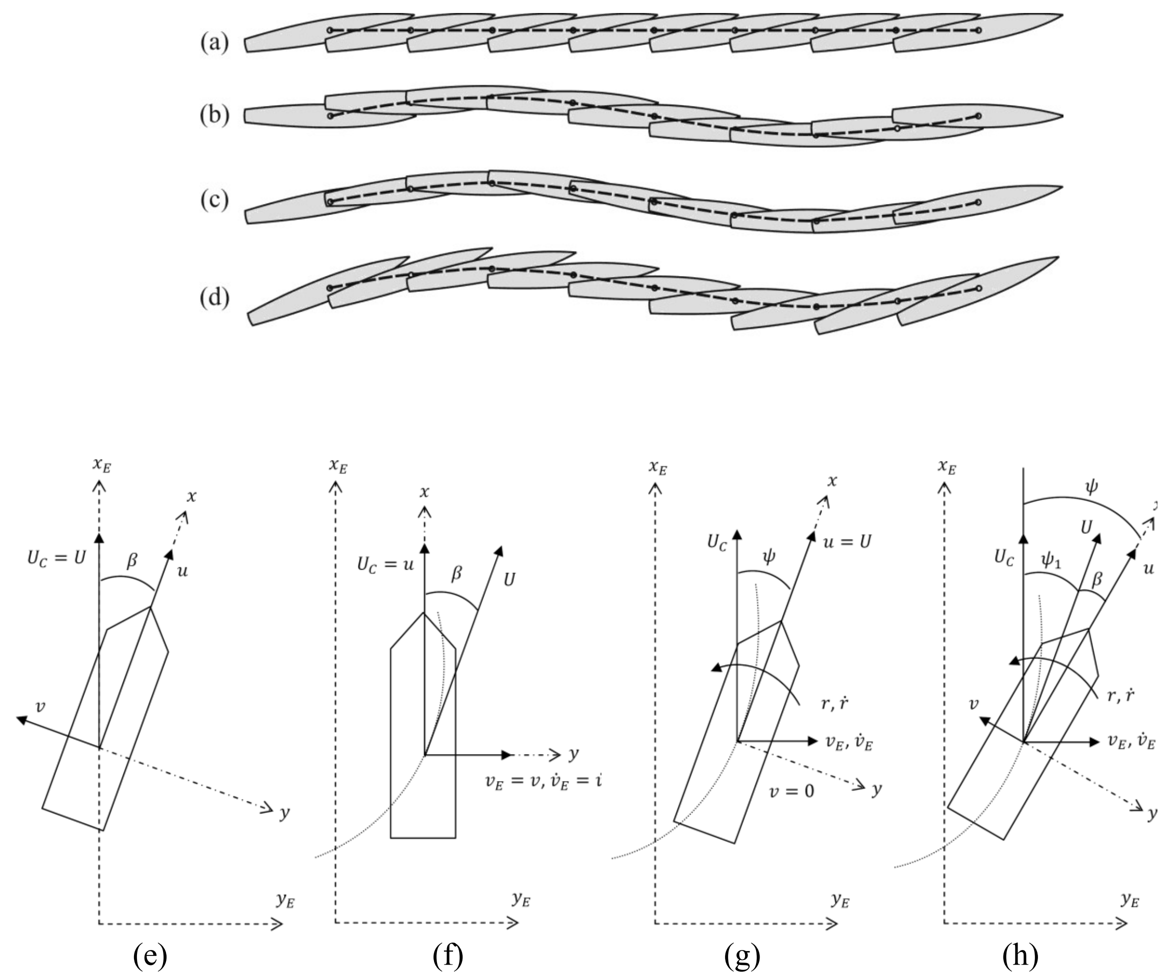

Figure 2-4 Illustrations of typical PMM motions for (a) static drift, (b) pure sway, (c) pure yaw, and (d) yaw and drift tests, and definitions of PMM motion parameters in the PMM coordinate systems for (e) static drift, (f) pure sway, (g) pure yaw, and (h) yaw and drift tests.

Table 2-1 PMM Motions in the Earth-fixed Coordinates.

Motion	Static drift	Pure sway	Pure yaw	Yaw and drift
u_E	U_C	U_C	U_C	U_C
$\dot{u}_E$	0	0	0	0
v_E	0	$-y_{max}\,\omega\cos\omega t$	$-y_{max}\,\omega\cos\omega t$	$-y_{max}\,\omega\cos\omega t$
$\dot{v}_E$	0	$y_{max}\,\omega^2\sin\omega t$	$y_{max}\,\omega^2\sin\omega t$	$y_{max}\,\omega^2\sin\omega t$
r_E	0	0	$\varepsilon\omega\sin\omega t\dfrac{1}{1+\varepsilon^2\cos^2\omega t}$	$\varepsilon\omega\sin\omega t\dfrac{1}{1+\varepsilon^2\cos^2\omega t}$
$\dot{r}_E$	0	0	$\varepsilon\omega^2\cos\omega t\dfrac{1+\varepsilon^2(1+\sin^2\omega t)}{(1+\varepsilon^2\cos^2\omega t)^2}$	$\varepsilon\omega^2\cos\omega t\dfrac{1+\varepsilon^2(1+\sin^2\omega t)}{(1+\varepsilon^2\cos^2\omega t)^2}$

Table 2-2 PMM Motions in the Ship-fixed Coordinates.

Motion	Static drift	Pure sway	Pure yaw	Yaw and drift
u	$U_C\cos\beta$	U_C	$U_C\sqrt{1+\varepsilon^2\cos^2\omega t}=u_1$	$u_1\cos\beta$
$\dot{u}$	0	0	$-U_C\omega\cdot\dfrac{\varepsilon^2\sin 2\omega t}{2\sqrt{1+\varepsilon^2\cos^2\omega t}}=\dot{u}_1$	$\dot{u}_1\cos\beta$
v	$-U_C\sin\beta$	$-y_{max}\,\omega\cos\omega t$	0	$-u_1\sin\beta$
$\dot{v}$	0	$y_{max}\,\omega^2\sin\omega t$	0	$-\dot{v}_1\sin\beta$
r	0	0	$\varepsilon\omega\sin\omega t\cdot\dfrac{1}{1+\varepsilon^2\cos^2\omega t}=r_1$	r_1
$\dot{r}$	0	0	$\varepsilon\omega^2\cos\omega t\cdot\dfrac{1+\varepsilon^2(1+\sin^2\omega t)}{(1+\varepsilon^2\cos^2\omega t)^2}=\dot{r}_1$	$\dot{r}_1$

The PMM motions, however, may violate the steady advance speed U condition for the Taylor-series expansions of hydrodynamic forces and moment shown in (2.4). If the surge $u_E=\dot{x}_E$ and sway $v_E=\dot{y}_E$ velocities from (2.8) and (2.9), respectively, are used in (2.5), then U becomes time-dependent (except for static drift case where $U=U_C$) and suggests PMM motions should be small such that

$$U=U_C\sqrt{1+\varepsilon^2\cos^2\omega t}=U_C+O(\varepsilon^2)\approx U_C\quad\text{for }\ \varepsilon\ll 1 \tag{2.12}$$

Then, the PMM motions summarized in Table 2-1 can be simplified as follows.

Static drift:

$$v = -U_C \sin\beta \tag{2.13}$$

Pure sway:

$$y = -y_{max}\ \sin\omega t \tag{2.14a}$$

$$v = -v_{max}\ \cos\omega t; \qquad v_{max}\ = y_{max}\,\omega \tag{2.14b}$$

$$\dot{v} = \dot{v}_{max}\ \sin\omega t; \qquad \dot{v}_{max}\ = y_{max}\,\omega^2 \tag{2.14c}$$

Then, drift angle β is from (6) as

$$\beta(t) = \beta_{max}\ \cos\omega t; \qquad \beta_{max}\ = \frac{y_{max}\ \omega}{U_C} \tag{2.15}$$

Pure yaw:

$$\psi = -\psi_{max}\ \cos\omega t; \qquad \psi_{max}\ = \frac{y_{max}\ \omega}{U_C} \tag{2.16a}$$

$$r = r_{max}\ \sin\omega t; \qquad r_{max}\ = \psi_{max}\,\omega \tag{2.16b}$$

$$\dot{r} = \dot{r}_{max}\ \cos\omega t; \qquad \dot{r}_{max}\ = \psi_{max}\,\omega^2 \tag{2.16c}$$

Yaw and drift:

$$\psi = -\psi_{max}\ \cos\omega t + \beta; \qquad \psi_{max}\ = \frac{y_{max}\ \omega}{U_C} \tag{2.17a}$$

$$v = -U_C \sin\beta \tag{2.17b}$$

where r and $\dot{r}$ for yaw and drift test are same as (2.16b) and (2.16c) for pure yaw test. For such small motions, i.e. $\varepsilon \ll 1$, and additionally for small β for static drift and yaw and drift tests, surge velocity $u \approx U_C$ and thus $\Delta u = u - U = 0$ for all tests.

2.3.3 Simplified Mathematic Models for PMM

For a bare model without propellers or rudders, the Abkowitz's mathematic models for hydrodynamic forces and moment shown in (2.4) can be reduced by dropping the terms related to rudder angle δ as:

$$X = X_* + X_{vv}v^2 + X_{rr}r^2 + X_{vr}vr$$
$$+X_u\Delta u + X_{uu}\Delta u^2 + X_{uuu}\Delta u^3 + X_{vvu}v^2\Delta u + X_{rru}r^2\Delta u + X_{vru}vr\Delta u \quad (2.18a)$$

$$Y = Y_{\dot{v}}\dot{v} + Y_v v + Y_{vvv}v^3 + Y_{\dot{r}}\dot{r} + Y_r r + Y_{rrr}r^3 + Y_{vrr}vr^2 + Y_{rvv}rv^2$$
$$+Y_{vu}v\Delta u + Y_{vuu}v\Delta u^2 + Y_{ru}r\Delta u + Y_{ruu}r\Delta u^2 \quad (2.18b)$$

$$N = N_{\dot{v}}\dot{v} + N_v v + N_{vvv}v^3 + N_{\dot{r}}\dot{r} + N_r r + N_{rrr}r^3 + N_{vrr}vr^2 + N_{rvv}rv^2$$
$$+N_{vu}v\Delta u + N_{vuu}v\Delta u^2 + N_{ru}r\Delta u + N_{ruu}r\Delta u^2 \quad (2.18c)$$

The math-models (18) are further simplified by using the simplified motions (2.13) – (2.17) to leave terms for the variables of interest and to determine the hydrodynamic derivatives.

Static drift:

$$X = X_* + X_{vv}v^2 \quad (2.19a)$$

$$Y = Y_v v + Y_{vvv}v^3 \quad (2.19b)$$

$$N = N_v v + N_{vvv}v^3 \quad (2.19c)$$

Pure sway:

$$X = X_* + X_{vv}v^2 \quad (2.20a)$$

$$Y = Y_{\dot{v}}\dot{v} + Y_v v + Y_{vvv}v^3 \quad (2.20b)$$

$$N = N_{\dot{v}}\dot{v} + N_v v + N_{vvv}v^3 \quad (2.20c)$$

or in harmonic forms by substituting (2.14b) and (2.14c) into (2.20),

$$X = X_0 + X_{C2} \cos 2\omega t \tag{2.21a}$$

$$Y = Y_{S1} \sin \omega t + Y_{C1} \cos \omega t + Y_{C3} \cos 3\omega t \tag{2.21b}$$

$$N = N_{S1} \sin \omega t + N_{C1} \cos \omega t + N_{C3} \cos 3\omega t \tag{2.21c}$$

Pure yaw:

$$X = X_* + X_{rr} r^2 \tag{2.22a}$$

$$Y = Y_{\dot{r}} \dot{r} + Y_r r + Y_{rrr} r^3 \tag{2.22b}$$

$$N = N_{\dot{r}} \dot{r} + N_r r + N_{rrr} v^3 \tag{2.22c}$$

or in harmonic forms by substituting (2.16b) and (2.16c) into (2.22),

$$X = X_0 + X_{C2} \cos 2\omega t \tag{2.23a}$$

$$Y = Y_{C1} \cos \omega t + Y_{S1} \sin \omega t + Y_{S3} \sin 3\omega t \tag{2.23b}$$

$$N = N_{C1} \cos \omega t + N_{S1} \sin \omega t + N_{S3} \sin 3\omega t \tag{2.23c}$$

Yaw and drift:

$$X = X_* + X_{vv} v^2 + X_{rr} r^2 + X_{vr} vr \tag{2.24a}$$

$$Y = Y_v v + Y_{vvv} v^3 + Y_{\dot{r}} \dot{r} + Y_r r + Y_{rrr} r^3 + Y_{vrr} vr^2 + Y_{rvv} rv^2 \tag{2.24b}$$

$$N = N_v v + N_{vvv} v^3 + N_{\dot{r}} \dot{r} + N_r r + N_{rrr} v^3 + N_{vrr} vr^2 + N_{rvv} rv^2 \tag{2.24c}$$

or in harmonic forms by substituting (2.16b), (2.16c), and (2.17b) into (2.24),

$$X = X_0 + X_{S1} \sin \omega t + X_{C2} \cos 2\omega t \tag{2.25a}$$

$$Y = Y_0 + Y_{C1} \cos \omega t + Y_{S1} \sin \omega t + Y_{C2} \cos 2\omega t + Y_{S3} \cos 3\omega t \tag{2.25b}$$

$$N = N_0 + N_{C1} \cos \omega t + N_{S1} \sin \omega t + N_{C2} \cos 2\omega t + N_{S3} \cos 3\omega t \tag{2.25c}$$

The expressions for the harmonics X_0, X_{Sn}, X_{Cn}, Y_0, Y_{Sn}, Y_{Cn}, N_0, N_{Sn}, and N_{Cn} for n = 1, 2, or 3 in (2.21), (2.23), and (2.25) are summarized in Table 2-3.

Table 2-3. Mathematic Models in Harmonics Forms.

Pure sway models:

$$X = X_0 + X_{C2} \cos 2\omega t$$

$$Y = Y_{C1} \cos \omega t + Y_{S1} \sin \omega t + Y_{C3} \cos 3\omega t$$

$$N = N_{C1} \cos \omega t + N_{S1} \sin \omega t + N_{C3} \cos 3\omega t$$

X model	*Y* model	*N* model
$X_0 = X_* + \frac{1}{2} X_{vv} v_{max}^2$	$Y_{C1} = -\left(Y_v v_{max} + \frac{3}{4} Y_{vvv} v_{max}^3\right)$	$N_{C1} = -\left(N_v v_{max} + \frac{3}{4} N_{vvv} v_{max}^3\right)$
$X_{C2} = \frac{1}{2} X_{vv} v_{max}^2$	$Y_{S1} = Y_{\dot{v}} \dot{v}_{max}$	$N_{S1} = N_{\dot{v}} \dot{v}_{max}$
	$Y_{C3} = -\frac{1}{4} Y_{vvv} v_{max}^3$	$N_{C3} = -\frac{1}{4} N_{vvv} v_{max}^3$

Pure yaw models:

$$X = X_0 + X_{C2} \cos 2\omega t$$

$$Y = Y_{S1} \sin \omega t + Y_{C1} \cos \omega t + Y_{S3} \sin 3\omega t$$

$$N = N_{S1} \sin \omega t + N_{C1} \cos \omega t + N_{S3} \sin 3\omega t$$

X model	*Y* model	*N* model
$X_0 = X_* + \frac{1}{2} X_{rr} r_{max}^2$	$Y_{S1} = Y_r r_{max} + \frac{3}{4} Y_{rrr} r_{max}^3$	$N_{S1} = N_r r_{max} + \frac{3}{4} N_{rrr} r_{max}^3$
$X_{C2} = -\frac{1}{2} X_{rr} r_{max}^2$	$Y_{C1} = Y_{\dot{r}} \dot{r}_{max}$	$N_{C1} = N_{\dot{r}} \dot{r}_{max}$
	$Y_{S3} = -\frac{1}{4} Y_{rrr} r_{max}^3$	$N_{S3} = -\frac{1}{4} N_{rrr} r_{max}^3$

Yaw and drift models:

$$X = X_0 + X_{S1} \sin \omega t + X_{C2} \cos 2\omega t$$

$$Y = Y_0 + Y_{S1} \sin \omega t + Y_{C1} \cos \omega t + Y_{C2} \cos 2\omega t + Y_{S3} \sin 3\omega t$$

$$N = N_0 + N_{S1} \sin \omega t + N_{C1} \cos \omega t + N_{C2} \cos 2\omega t + N_{S3} \sin 3\omega t$$

X model	*Y* model	*N* model
$X_0 = X_* + X_{vv} v^2 + \frac{1}{2} X_{rr} r_{max}^2$	$Y_0 = Y_v v + Y_{vvv} v^3 + \frac{1}{2} Y_{vrr} v r_{max}^2$	$N_0 = N_v v + N_{vvv} v^3 + \frac{1}{2} N_{vrr} v r_{max}^2$
$X_{S1} = X_{vr} v r_{max}$	$Y_{S1} = Y_r r_{max} + \frac{3}{4} Y_{rrr} r_{max}^3 + Y_{rvv} r_{max} v^2$	$N_{S1} = N_r r_{max} + \frac{3}{4} N_{rrr} r_{max}^3 + N_{rvv} r_{max} v^2$
$X_{C2} = -\frac{1}{2} X_{rr} r_{max}^2$	$Y_{C1} = Y_{\dot{r}} \dot{r}_{max}$	$N_{C1} = N_{\dot{r}} \dot{r}_{max}$
	$Y_{C2} = -\frac{1}{2} Y_{vrr} v r_{max}^2$	$N_{C2} = -\frac{1}{2} N_{vrr} v r_{max}^2$
	$Y_{S3} = -\frac{1}{4} Y_{rrr} r_{max}^3$	$N_{S3} = -\frac{1}{4} N_{rrr} r_{max}^3$

2.3.4 Non-dimensionalization

Non-dimensionlization follows the *Prime-system* of SNAME (Nomenclature, 1952) for which L, L/U, and ½$\rho L^2 T$ are used as the characteristic scales for length, time, and mass, respectively, where L is the ship length, U is the ship advance speed, ρ is the water density, and T is the draft of the ship. Some of the non-dimensional variables are shown below:

$$y = \frac{y}{L}; \qquad y'_{max} = \frac{y_{max}}{L} \tag{2.26a}$$

$$\omega' = \frac{\omega L}{U} \approx \frac{\omega L}{U_C} \tag{2.26b}$$

$$\Delta u' = u' - 1 = \frac{u}{U} - 1 \tag{2.26c}$$

$$\dot{u}' = \frac{\dot{u} L}{U^2} \tag{2.26d}$$

$$v' = \frac{v}{U}; \qquad v'_{max} = \frac{v_{max}}{U} \approx \left(\frac{y_{max}}{L}\right)\left(\frac{\omega L}{U_C}\right) \tag{2.26e}$$

$$\dot{v}' = \frac{\dot{v} L}{U^2}; \qquad \dot{v}'_{max} = \frac{\dot{v}_{max} L}{U^2} \approx \left(\frac{y_{max}}{L}\right)\left(\frac{\omega L}{U_C}\right)^2 \tag{2.26f}$$

$$r' = \frac{rL}{U}; \qquad r'_{max} = \frac{r_{max} L}{U} \approx \psi_{max}\left(\frac{\omega L}{U_C}\right) \tag{2.26g}$$

$$\dot{r}' = \frac{\dot{r} L^2}{U^2}; \qquad \dot{r}'_{max} = \frac{\dot{r}_{max} L^2}{U^2} \approx \psi_{max}\left(\frac{\omega L}{U_C}\right)^2 \tag{2.26h}$$

$$X' = \frac{X}{1/2\rho U^2 LT} \tag{2.26i}$$

$$Y' = \frac{Y}{1/2\rho U^2 LT} \tag{2.26j}$$

$$N' = \frac{N}{1/2\rho U^2 L^2 T} \tag{2.26k}$$

Note that in the remainder of the thesis the prime symbol is omitted for simplicity.

2.3.5 Determination of hydrodynamic derivatives

Hydrodynamic derivatives (simply 'derivatives') in the mathematic models (2.18) are determined from the static drift, pure sway, pure yaw, and yaw and drift data. Sway-velocity derivatives X_*, X_{vv}, Y_v, Y_{vvv}, N_v, and N_{vvv} are determined from the static drift data and sway-acceleration derivatives $Y_{\dot{v}}$ and $N_{\dot{v}}$ are from the pure sway data. Sway-velocity derivatives can be determined as well from the pure sway data, however, derivatives determined from the static drift data are preferred in general as the derivatives from dynamic-test data are known as often frequency-dependent (van Leeuwen 1964). As the dynamic-motion frequency ω becomes large, the 'quasi-steady' or the 'slow-motion' assumptions for the math-models can fail and the hydrodynamic forces and moment during the PMM tests become dependent not only on the instantaneous motions but partly also on the previous motions (Bishop et al. 1970, 1972, 1973), known as the 'memory effect'. The yaw-rate derivatives X_{rr}, Y_r, Y_{rrr}, N_r, and N_{rrr} and the yaw-acceleration derivatives $Y_{\dot{r}}$ and $N_{\dot{r}}$ are determined from the pure yaw test. The cross-coupled derivatives between sway and yaw such as X_{vr}, Y_{vrr}, Y_{rvv}, N_{vrr}, and N_{rvv} are determined from the yaw and drift test that is a combination of pure yaw and static drift tests. The surge-coupled derivatives such as X_u, X_{uu}, X_{uuu}, X_{vvu}, Y_{vu}, Y_{vuu}, N_{vu}, and N_{vuu} are determined by repeating the static drift (or pure sway) test and X_{rru}, Y_{ru}, Y_{ruu}, N_{ru}, N_{ruu} are by repeating the pure yaw test over a range of towing speed, respectively. The sway-yaw-surge-coupled derivative X_{vru} can be determined by repeating the yaw and drift test, but typically of negligible value.

The derivatives are evaluated by curve-fitting the data for static drift test and by using either the 'Multiple-run (MR)' or 'Single-run (SR)' methods for dynamic tests as per introduced below:

2.3.5.1 Static drift test

Data are measured over a range of drift angle β and curve-fitted to polynomial functions as per the mathematic model (2.19):

$$y = A + Bx^2;\ y = X;\ x = v \tag{2.27a}$$

$$y = Ax + Bx^3;\ y = Y, N;\ x = v \tag{2.27b}$$

Then,

$$X_*, Y_v, N_v = A \tag{2.28a}$$

$$X_{vv}, Y_{vvv}, N_{vvv} = B \tag{2.28b}$$

respectively.

2.3.5.2 Dynamic tests

Derivatives can be determined from the math-models (2.20), (2.22), and (2.24) with expressed in harmonics form, summarized in Table 2-3. Then, the derivatives[5] are evaluated either by curve-fitting the harmonics data into those equations, named as the '*Multiple-Run*' method; or by solving the harmonics equations for the derivatives, named as the '*Single-Run*' method. The harmonics data are determined experimentally by measuring the X, Y, and N as time-histories from PMM tests as

$$X, Y, N = f(t) \tag{2.29}$$

and using a Fourier-integral equation as:

[5] Derivatives can be also determined by using a regression method, although not used herein. By using the math-models (2.19) as the regression equations, the PMM test data can be curve-fitted using such as a Least-square-error method to evaluate the derivatives.

$$X_0, Y_0, N_0 = \frac{1}{T}\int_0^T f(t)dt \tag{2.30a}$$

$$X_{Cn}, Y_{Cn}, N_{Cn} = \frac{2}{T}\int_0^T f(t)\cos n\omega t\, dt \tag{2.30b}$$

$$X_{Sn}, Y_{Sn}, N_{Sn} = \frac{2}{T}\int_0^T f(t)\sin n\omega t\, dt \tag{2.30c}$$

where $T = 2\pi/\omega$.

'Multiple-run' (MR) method: Derivatives are determined by using data from a series of PMM tests. For this, PMM tests are repeated over a range of input motions parameters such as v_{max}, r_{max}, or β, and then a set of harmonics data, evaluated from each test as per (2.30), is fitted into polynomial functions as

$$\left.\begin{matrix} X_0, Y_0, N_0 \\ X_{Sn}, Y_{Sn}, N_{Sn} \\ X_{Cn}, Y_{Cn}, N_{Cn} \end{matrix}\right\} = y(x);\ \ x = v_{max}, \dot{v}_{max}, r_{max}, \dot{r}_{max}, \text{ or } v \tag{2.31}$$

Polynomial functions $y(x)$ in (2.31) for each harmonic are summarized in Table 2-4 where the resulting hydrodynamic derivatives are expressed with the polynomial coefficients. From Table 2-4, the non-linear derivatives such as X_{vv}, Y_{vvv}, N_{vvv}, X_{rr}, Y_{rrr}, N_{rrr}, Y_{vrr}, and N_{vrr} can be determined either from the 0^{th}- or 1^{st}-order (low-order) harmonics such as X_0, Y_0, Y_{C1}, Y_{S1}, N_0, N_{S1}, and N_{S1} or from the 2^{nd}- or 3^{rd}-order (high-order) harmonics such as X_{C2}, Y_{C2}, Y_{C3}, Y_{S3}, N_{C2}, N_{C3}, or N_{S3}, which are designated as the 'MR_L' and the 'MR_H' methods, respectively.

'Single-run' (SR) method: Hydrodynamic derivatives are determined by using data from a single realization (carriage-run) of dynamic PMM test (or from a mean-data by repeating the tests at the same condition). First, FS harmonics of the data are evaluated as per (2.30), and then the equations of harmonics amplitudes in Table 2-3 are solved for hydrodynamic derivatives such that

$$\left.\begin{matrix} X_{*}, X_{vv}, X_{rr}, X_{vr}; \\ Y_{v}, Y_{vvv}, Y_{\dot{v}}, Y_{r}, Y_{rrr}, Y_{\dot{r}}, Y_{vrr}, Y_{rvv}; \\ N_{v}, N_{vvv}, N_{\dot{v}}, N_{r}, N_{rrr}, N_{\dot{r}}, N_{vrr}, N_{rvv} \end{matrix}\right\} = f\begin{pmatrix} X_0, X_{Sn}, \text{or } X_{Cn}; \\ Y_0, Y_{Sn}, \text{or } Y_{Cn}; \\ N_0, N_{Sn}, \text{or } N_{Cn} \end{pmatrix} \tag{2.32}$$

respectively, where n = 1, 2, or 3. The solutions are summarized in Table 2-5, where two derivatives, Y_{vrr} and N_{vrr}, can be determined either from the 0^{th}-order (low-order) harmonics Y_0 and N_0 or from the 2^{nd}-order (high-order) harmonics Y_{C2} and N_{C2}, which are designated as the 'SR_L' and the 'SR_H' methods, respectively.

Table 2-4. '*Multiple-Run*' Method.

Test	Variable	Polynomial equation	y	x	Derivatives
Pure sway	X	$y = A + Bx^2$	X_0	v_{max}	$X_* = A; X_{vv} = 2B$
		$y = Cx^2$	X_{C2}	v_{max}	$X_{vv} = 2C$
	Y, N	$y = Ax + Bx^3$	Y_{C1}, N_{C1}	v_{max}	$Y_v, N_v = A; Y_{vvv}, N_{vvv} = \frac{4}{3}B$
		$y = Cx$	Y_{S1}, N_{S1}	$\dot{v}_{max}$	$Y_{\dot{v}}, N_{\dot{v}} = C$
`		$y = Dx^3$	Y_{C3}, N_{C3}	v_{max}	$Y_{vvv}, N_{vvv} = -4D$
Pure yaw	X	$y = A + Bx^2$	X_0	r_{max}	$X_* = A; X_{rr} = 2B$
		$y = Cx^2$	X_{C2}	r_{max}	$X_{rr} = -2C$
	Y, N	$y = Ax + Bx^3$	Y_{S1}, N_{S1}	r_{max}	$Y_r, N_r = A; Y_{rrr}, N_{rrr} = \frac{4}{3}B$
		$y = Cx$	Y_{C1}, N_{C1}	$\dot{r}_{max}$	$Y_{\dot{r}}, N_{\dot{r}} = C$
		$y = Dx^3$	Y_{S3}, N_{S3}	r_{max}	$Y_{rrr}, N_{rrr} = 4D$
Yaw and drift	X	$y = Ax$	X_{C1}	v	$X_{vr} = \frac{1}{r_{max}}A$
	Y, N	$y = Ax + Bx^3$	Y_0, N_0	v	$Y_{vrr} = \frac{2}{r_{max}^2}(A - Y_v); N_{vrr} = \frac{2}{r_{max}^2}(A - N_v)$
		$y = C + Dx^2$	Y_{S1}, N_{S1}	v	$Y_{rvv}, N_{rvv} = \frac{1}{r_{max}}D$
		$y = Ex$	Y_{C2}, N_{C2}	v	$Y_{vrr}, N_{vrr} = -\frac{2}{r_{max}^2}E$

Table 2-5. '*Single-Run*' Method.

Pure sway	Pure yaw	Yaw and drift
$X_* = X_0 - X_{C2}$	$X_* = X_0 + X_{C2}$	
$Y_v = -\frac{1}{v_{max}}(Y_{C1} - 3Y_{C3})$	$Y_r = \frac{1}{r_{max}}(Y_{S1} + 3Y_{S3})$	
$N_v = -\frac{1}{v_{max}}(N_{C1} - 3N_{C3})$	$N_r = \frac{1}{r_{max}}(N_{S1} + 3N_{S3})$	
$X_{vv} = \frac{2}{v_{max}^2}X_{C2}$	$X_{rr} = -\frac{2}{r_{max}^2}X_{C2}$	$X_{vr} = \frac{1}{vr_{max}}X_{S1}$
$Y_{vvv} = -\frac{4}{v_{max}^3}Y_{C3}$	$Y_{rrr} = -\frac{4}{r_{max}^3}Y_{S3}$	$Y_{vrr} = \frac{2}{vr_{max}^2}(Y_0 - Y_v v - Y_{vvv}v^3)$ or $-\frac{2}{vr_{max}^2}Y_{C2}$
$N_{rrr} = -\frac{4}{v_{max}^3}N_{C3}$	$N_{rrr} = -\frac{4}{r_{max}^3}N_{S3}$	$N_{vrr} = \frac{2}{vr_{max}^2}(N_0 - N_v v - N_{vvv}v^3)$ or $-\frac{2}{vr_{max}^2}N_{C2}$
$Y_{\dot{v}} = \frac{1}{\dot{v}_{max}}Y_{S1}$	$Y_{\dot{r}} = \frac{1}{\dot{r}_{max}}Y_{C1}$	$Y_{rvv} = \frac{1}{r_{max}\,v^2}\left(Y_{S1} - Y_r r_{max} - \frac{3}{4}Y_{rrr}r_{max}^3\right)$
$N_{\dot{v}} = \frac{1}{\dot{v}_{max}}N_{S1}$	$Y_{\dot{r}} = \frac{1}{\dot{r}_{max}}N_{C1}$	$N_{rvv} = \frac{1}{r_{max}\,v^2}\left(N_{S1} - N_r r_{max} - \frac{3}{4}N_{rrr}r_{max}^3\right)$

2.3.5.3 Speed variation test

Surge-derivatives such as X_u, X_{uu}, and X_{uuu} in (2.18) are determined by repeating the static drift test at the $\beta = 0$ for a range of U (i.e. U_C). The static drift X at $\beta = 0$, the steady reference state value X_*, corresponds to the resistance of the model at the speed U as no propeller is working. If the model towing speed is changed, say $U + \Delta u$, the X_* value will change as the model resistance increase (or decrease) such that

$$X_*(U + \Delta u) = X_*(U) + \Delta X \tag{2.33}$$

The changes in resistance ΔX in (2.33) can be written using a Taylor series expansion as

$$\Delta X = f(u) = \frac{\partial X}{\partial u}\Delta u + \frac{1}{2}\frac{\partial^2 X}{\partial u^2}\Delta u^2 + \frac{1}{6}\frac{\partial^3 X}{\partial u^3}\Delta u^3 + \cdots \tag{2.34}$$

where the differentiations of X are evaluated at $\Delta u = 0$ or $u = U$, which are identical with the definitions of surge hydrodynamic derivatives. When the test is repeated over a range of U, the measured X values can be expressed as a polynomial function of $\Delta u = u - U$ as

$$f(u) = a_0 + a_1\Delta u + a_2\Delta u^2 + a_3\Delta u^3 + \cdots \tag{2.35}$$

and hydrodynamic derivatives X_u, X_{uu}, X_{uuu} are determined as following:

$$X_u = \frac{\partial f}{\partial u} = a_1 \tag{2.36a}$$

$$X_{uu} = \frac{1}{2}\frac{\partial^2 f}{\partial u^2} = a_2 \tag{2.36b}$$

$$X_{uuu} = \frac{1}{6}\frac{\partial^3 f}{\partial u^3} = a_3 \tag{2.36c}$$

Derivatives such as X_{vv}, Y_v, N_v, and X_{rr}, Y_r, N_r, and X_{vr} evaluated at U may also change with Δu and can be expressed as appropriate polynomial functions $f(u)$ similarly as

(2.35) by repeating the static drift tests, pure yaw test, and yaw and drift test, respectively. Subsequently, the surge-coupled hydrodynamic derivatives such as X_{vvu}, X_{rru}, X_{vru}, Y_{vu}, Y_{vuu}, N_{vu}, N_{vuu} are determined as following:

$$\left.\begin{matrix} X_{vvu}, X_{rru}, X_{vru} \\ Y_{vu}, Y_{ru} \\ N_{vu}, N_{ru} \end{matrix}\right\} = \frac{\partial f}{\partial u} = a_1 \tag{2.37a}$$

$$\left.\begin{matrix} Y_{vuu}, Y_{ruu} \\ N_{vuu}, N_{ruu} \end{matrix}\right\} = \frac{1}{2}\frac{\partial^2 f}{\partial u^2} = a_2 \tag{2.37b}$$