Example: A small, axisymmetric, low-speed wind tunnel is built to calibrate hot wires. As the boundary layer grows along the wall of the wind tunnel test section, air in the region of irrotational flow in the central portion of the test section accelerates in order to satisfy conservation of mass. To eliminate this acceleration, the engineers can diverge the test section walls. Assume laminar flow, the wind tunnel diverge linearly, the diameter of the test section inlet is 6.0 in, and its length is 10.0 in. The air is at 70°F and enters the wind tunnel at a uniform air speed of 5.0 ft/s at the test section inlet. What is the diameter at the test section outlet?

Solution:

The kinematic viscosity of air at 70°F is $\nu = 1.643\times10^{-4}\ ft^2/s$

For incompressible flow, $Q_0 = Q(x)$, where $Q_0 =$ flowrate into the tunnel.

$Q_0 = UA_0 = U\frac{1}{4}\pi d_0^2$, and

$Q(x) = UA$, where $A = \frac{1}{4}\pi\left(d - 2\delta^*\right)^2$ is the effective area of the tunnel (allowing for the decreased flowrate in the boundary layer)

Thus

$Q_0 = U\frac{1}{4}\pi d_0^2 = U\frac{1}{4}\pi\left(d - 2\delta^*\right)^2$ or $d = d_0 + 2\delta^*$

Where $\delta^* = 1.721\sqrt{\frac{\nu x}{U}} = 1.721\sqrt{\frac{\nu x}{U}} = 1.721\sqrt{\frac{1.643\times10^{-4}x}{5}} = 0.009865\sqrt{x}$

So $d = d_0 + 0.01973\sqrt{x}$

At the outlet of the wind tunnel, $x = 10in = \frac{10}{12}ft$, so

$$d = d_0 + 0.01973\sqrt{x} = \frac{6}{12} + 0.01973\sqrt{\frac{10}{12}} = 0.518\,ft$$