10.24 The velocity of oil (S = 0.8) through the 2-in. smooth pipe is 5 ft/s. Here $L = 30$ ft, $z_1 = 2$ ft, $z_2 = 4$ ft, and the manometer deflection is 4 in. Determine the flow direction, the resistance coefficient f, whether the flow is laminar or turbulent, and the viscosity of the oil.

10.24

$$V(r) = \frac{r_0^2 - r^2}{4\mu}\left[-\frac{d}{ds}(p+\gamma z)\right]$$ continuity + momentum

favorable pressure gradient $\frac{d}{ds}(p+\gamma z) < 0$

adverse pressure gradient $\frac{d}{ds}(p+\gamma z) > 0$

$h_L = -\Delta h = h_1 - h_2$ $\quad h_1 = \frac{p_1}{\gamma} + z_1$ $\quad h_2 = \frac{p_2}{\gamma} + z_1$ energy

$$h_f = h_L = \frac{L}{\gamma}\underbrace{\left[-\frac{d}{ds}(p+\gamma z)\right]}_{8\mu\bar{V}/r_0^2}$$ from definition $\bar{V}$

$$h_f = \frac{32\mu L\bar{V}}{\gamma D^2}$$ exact solution laminar pipe flow

Also $\frac{L}{\gamma}\left[-\frac{d}{ds}(p+\gamma z)\right] = -\Delta h$

$$-\frac{d}{ds}(p+\gamma z) = \frac{\gamma}{L}(-\Delta h) \qquad \Delta h < 0 \quad \text{fav}$$

$$\frac{d}{ds}(p+\gamma z) = \frac{\gamma}{L}(\Delta h) \qquad \Delta h > 0 \quad \text{adv}$$

$$h_L = -\Delta h = \frac{p_1 - p_2}{\gamma} + (z_1 - z_2)$$

$$= -\frac{1}{\gamma}\left[p_1 - p_2 + (z_1 - z_2)\gamma\right]$$

$$= \frac{L}{\gamma}\left[-\frac{d}{ds}(p+\gamma z)\right]$$

Use manometer to determine sign Δh & flow direction

$$p_1 + \gamma z_1 + \gamma_m d - \gamma d - \gamma z_2 = p_2$$

$$\left(\frac{p_1}{\gamma} + z_1\right) - \left(\frac{p_2}{\gamma} + z_2\right) = -\Delta h = \frac{\gamma - \gamma_m}{\gamma} d$$

$$\Delta h = \frac{\gamma_m - \gamma}{\gamma} d$$

$$h_f = \frac{32 \mu L \bar{V}}{\gamma D^2}$$

$$= \frac{S_m - S}{S} \cdot d$$

$$\mu = \frac{\gamma D^2 h_f}{32 L \bar{V}}$$

$$= \frac{13.55 - .8}{.8} \times 4/12$$

$$= \frac{.8 \times 62.4 \times (2/12)^2 \times 5.312}{32 \times 30 \times 5}$$

$$= 5.312' > 0$$

$\therefore$ adv

$$= 1.53 \times 10^{-3} \text{ lb-sec/ft}^2$$

$\therefore$ flow from right to left

$$Re = \frac{V D \rho}{\mu} = \frac{5 \times (2/12) \times 1.55}{1.53 \times 10^{-3}}$$

$$\gamma = \rho g = .8 \times 62.4$$

$$\rho = \frac{.8 \times 62.4}{32.2}$$

$$= 844 \quad \text{OK laminar}$$

$$= 1.55 \frac{\text{slug}}{\text{ft}^3}$$