10–21 A drop of water in a rain cloud has diameter $D = 30\ \mu\text{m}$ (Fig. P10–21). The air temperature is 25°C, and its pressure is standard atmospheric pressure. How fast does the air have to move vertically so that the drop will remain suspended in the air? *Answer:* 0.0264 m/s

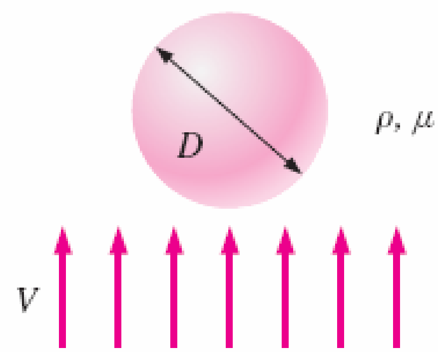

FIGURE P10–21

10-21

Solution We are to calculate how fast air must move vertically to keep a water drop suspended in the air.

Assumptions **1** The drop is spherical. **2** The creeping flow approximation is appropriate.

Properties For air at $T = 25°C$, $\rho = 1.184$ kg/m^3 and $\mu = 1.849 \times 10^{-5}$ kg/m·s. The density of the water at $T = 25°C$ is 997.0 kg/m^3.

Analysis Since the drop is sitting still, its downward force must exactly balance its upward force when the vertical air speed V is "just right". The downward force is the weight of the particle:

Downward force on the particle:

$$F_{\text{down}} = \pi \frac{D^3}{6} \rho_{\text{particle}} g \qquad (1)$$

The upward force is the aerodynamic drag force acting on the particle plus the buoyancy force on the particle. The aerodynamic drag force is obtained from the creeping flow drag on a sphere,

Upward force on the particle:

$$F_{\text{up}} = 3\pi\mu VD + \pi \frac{D^3}{6} \rho_{\text{air}} g \qquad (2)$$

We equate Eqs. 1 and 2, i.e., $F_{\text{down}} = F_{\text{up}}$,

Balance:

$$\pi \frac{D^3}{6} \left(\rho_{\text{particle}} - \rho_{\text{air}} \right) g = 3\pi\mu VD$$

and solve for the required air speed V,

$$V = \frac{D^2}{18\mu} \left(\rho_{\text{particle}} - \rho_{\text{air}} \right) g$$

$$= \frac{\left(30 \times 10^{-6} \text{ m}\right)^2}{18\left(1.849 \times 10^{-5} \text{ kg/m} \cdot \text{s}\right)} \left[\left(998.0 - 1.184\right) \text{ kg/m}^3 \right] \left(9.81 \text{ m/s}^2\right) = \mathbf{0.0264 \text{ m/s}}$$

Finally, we must verify that the Reynolds number is small enough that the creeping flow approximation is appropriate.

Check of Reynolds number:

$$\text{Re} = \frac{\rho_{\text{air}} VD}{\mu} = \frac{\left(1.184 \text{ kg/m}^3\right)\left(0.0264 \text{ m/s}\right)\left(30 \times 10^{-6} \text{ m}\right)}{1.849 \times 10^{-5} \text{ kg/m} \cdot \text{s}} = \mathbf{0.0507}$$

Since Re << 1, The creeping flow approximation is appropriate.

Discussion Notice that although air density does appear in the calculation of V, it is very small compared to the density of water. (If we ignore ρ_{air} in that calculation, we get V = 0.0265 m/s, an error of less than 0.4%. However, ρ_{air} *is* required in the calculation of Reynolds number – to verify that the creeping flow approximation is appropriate.