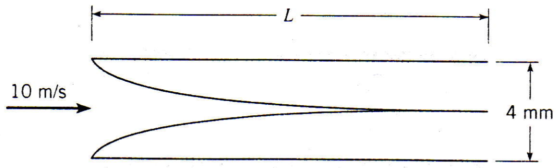
9.67 A model is being developed for the entrance region between two flat plates. As shown in the figure, it is assumed that the region is approximated by a turbulent boundary layer originating at the leading edge. The system is designed such that the plates end where the boundary layers merge. The spacing between the plates is 4 mm and the entrance velocity is 10 m/s. The fluid is water at 20°C. Roughness at the leading edge trips the boundary layers. Find the length L where the boundary layers merge and find the force per unit depth (into the paper) due to shear stress on both plates.



PROBLEM 9.67

9.67 **Information and assumptions**

provided in problem statement

Find

length where boundary layers merge and the shearing force per unit depth.

Solution

The density and kinematic viscosity of water at these conditions are 1000 kg/m^3 and 10^{-6} m^2/s. The boundary layer growth is given by

$$\delta = \frac{0.37x}{\mathrm{Re}_x^{1/5}} = \frac{0.37x^{4/5}}{\left(\frac{U_o}{\nu}\right)^{1/5}}$$

Setting $\delta = 0.002$ m and $x = L$, we have

$$L^{4/5} = \frac{0.002}{0.37}\left(\frac{10}{10^{-6}}\right)^{1/5} = 0.135$$

or

$$L = \underline{\underline{0.0824 \text{ m}}}$$

Check the Reynolds number

$$\mathrm{Re}_x = \frac{0.0824 \times 10}{10^{-6}} = 8.24 \times 10^5$$

so the equations for the tripped boundary layer ($Re_x < 10^7$) are valid.
The average shear stress coefficient is

$$C_f = \frac{0.074}{\left(\frac{0.0824\times 10}{10^{-6}}\right)^{1/5}} = 0.00485$$

The force due to shear stress on both plates is

$$\frac{F_s}{B} = 2 \times \frac{1}{2}\rho U_o^2 C_f L = 1000 \times 10^2 \times 0.00485 \times 0.0824 = \underline{\underline{40.0 \text{ N/m}}}$$