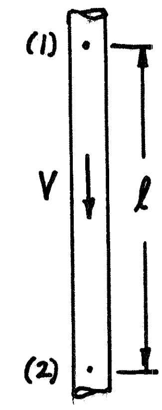
8.83

8.83 Water flows downward through a vertical smooth pipe. When the flowrate is 0.5 ft^3/s there is no change in pressure along the pipe. Determine the diameter of the pipe.



$$\frac{p_1}{\gamma}+z_1+\frac{V_1^2}{2g}=\frac{p_2}{\gamma}+z_2+\frac{V_2^2}{2g}+f\frac{\ell}{D}\frac{V^2}{2g}$$

where $p_1=p_2$, $V_1=V_2=V$, and $z_1-z_2=\ell$

Thus,

$$\ell=f\frac{\ell}{D}\frac{V^2}{2g}\text{, or } 1=\frac{f}{D}\frac{V^2}{2g} \qquad (1)$$

Also,

$V=\frac{Q}{A}=\frac{Q}{\frac{\pi}{4}D^2}$ so that Eq. (1) becomes $1=\frac{f}{D}\frac{\left(\frac{4Q}{\pi D^2}\right)^2}{2g}$

or

$$D^5=\frac{8}{\pi^2}f\frac{Q^2}{g}=\frac{8}{\pi^2}f\frac{(0.5)^2}{32.2} \text{ or } D=0.363\,f^{1/5} \qquad (2)$$

Also,

$$Re=\frac{\rho VD}{\mu}=\frac{1.94\left(\frac{4Q}{\pi D^2}\right)D}{\mu}=\frac{1.94\left(\frac{4(0.5)}{\pi}\right)}{2.34\times10^{-5}\,D} \text{ or } Re=\frac{5.28\times10^4}{D} \qquad (3)$$

From Fig. 8.20 with $\frac{\varepsilon}{D}=0$ we have $f=f(Re,\frac{\varepsilon}{D}=0)$

Trial and error solution: 3 unknowns (D, Re, f) and 3 equations ((2), (3), and Fig. 8.20)

Assume $f=0.02$ so from Eq. (2), $D=0.166$ ft and from Eq. (3), $Re=3.18\times10^5$. Thus, from Fig. 8.20, $f=0.014\neq0.02$

Assume $f=0.014$ so that $D=0.155$ ft and $Re=3.42\times10^4$. Thus, from Fig. 8.20, $f\approx0.14$ which checks with the assumed value.

Thus, $D=\underline{\underline{0.155\text{ ft}}}$

An alternative method is to use the Colebrook equation, Eq. 8.35, with $\varepsilon/D=0$, rather than the Moody chart, Fig. 8.20. Thus, $\frac{1}{\sqrt{f}}=-2.0\log(2.51/Re\sqrt{f})$ which combined with Eqs. (2) and (3) gives

$$1/(D/0.363)^{5/2}=-2.0\log\left[2.51D/(5.28\times10^4(D/0.363)^{5/2})\right] \qquad (4)$$

Using a computer root-finding program gives the solution of Eq. (4) as $D=0.155$ ft as obtaing by the above trial and error method.