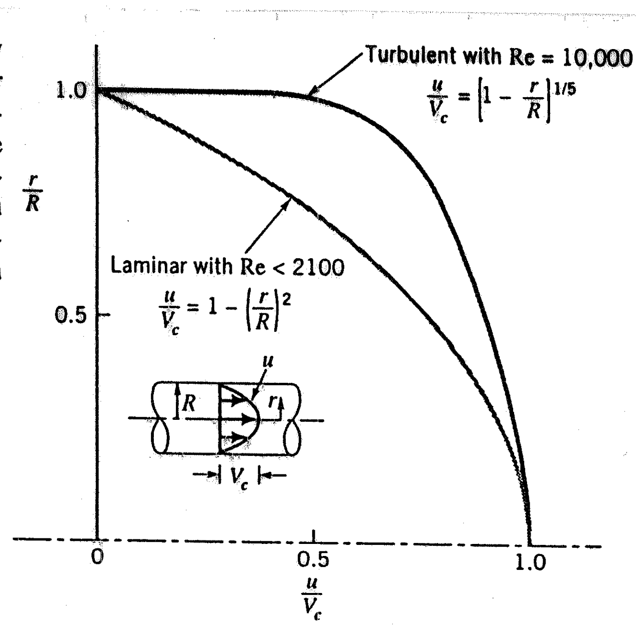
8.26

8.26 As shown in Video V8.3 and Fig. P8.26, the velocity profile for laminar flow in a pipe is quite different from that for turbulent flow. With laminar flow the velocity profile is parabolic; with turbulent flow at Re = 10,000 the velocity profile can be approximated by the power-law profile shown in the figure. (a) For laminar flow, determine at what radial loaction you would place a Pitot tube if it is to measure the average velocity in the pipe. (b) Repeat part (a) for turbulent flow with Re = 10,000.



■ FIGURE P8.26

For laminar or turbulent flow,

$$Q = AV = \pi R^2 V = \int u\,dA = \int u(2\pi r\,dr) = 2\pi \int_{r=0}^{R} u\,r\,dr$$

a) Laminar flow:

$$\pi R^2 V = 2\pi V_c \int_0^R r\left[1 - \left(\tfrac{r}{R}\right)^2\right]dr = 2\pi V_c\left[\frac{R^2}{2} - \frac{R^2}{4}\right] = \pi \frac{R^2}{2} V_c$$

Thus, $V = \frac{1}{2}V_c$ For $u = V = \frac{V_c}{2}$ the equation for $\frac{u}{V_c}$ gives

$$\frac{u}{V_c} = \frac{1}{2} = 1 - \left(\tfrac{r}{R}\right)^2, \text{ or } \left(\tfrac{r}{R}\right)^2 = \frac{1}{2} \quad \text{Thus, } r = \frac{1}{\sqrt{2}}R = \underline{\underline{0.707R}}$$

b) Turbulent flow

$$\pi R^2 V = 2\pi V_c \int_0^R r\left[1 - \tfrac{r}{R}\right]^{1/5} dr = 2\pi R^2 V_c \int_0^1 \left(\tfrac{r}{R}\right)\left[1 - \left(\tfrac{r}{R}\right)\right]^{1/5} d\left(\tfrac{r}{R}\right)$$

Let $y \equiv 1 - \left(\frac{r}{R}\right)$ so that $\left(\frac{r}{R}\right) = 1 - y$ and $d\left(\frac{r}{R}\right) = -dy$

Thus,

$$\pi R^2 V = 2\pi R^2 V_c \int_{y=1}^{y=0} (1-y)\,y^{1/5}(-dy) = 2\pi R^2 V_c \int_0^1 \left(y^{1/5} - y^{6/5}\right)dy$$

$$= 2\pi R^2 V_c\left[\frac{5}{6} - \frac{5}{11}\right] = 2\pi R^2 V_c\left(\frac{25}{66}\right)$$

or $V = \frac{50}{66}V_c$ For $u = V = \frac{50}{60}$ the equation for $\frac{u}{V_c}$ gives

$$\frac{u}{V_c} = \frac{50}{66} = \left[1 - \frac{r}{R}\right]^{1/5} \quad \text{or} \quad \frac{r}{R} = 0.750 \text{ so that } r = \underline{\underline{0.750\,R}}$$