8.22 Oil of $SG = 0.87$ and a kinematic viscosity $\nu = 2.2 \times 10^{-4}\ m^2/s$ flows through the vertical pipe shown in Fig. P8.22 at a rate of $4 \times 10^{-4}\ m^3/s$. Determine the manometer reading, h.

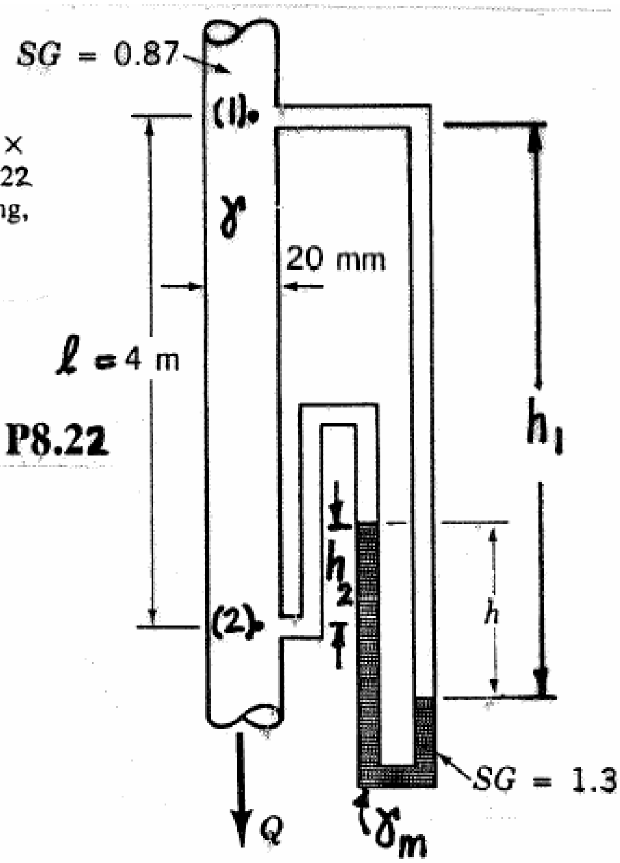


FIGURE P8.22

$$V = \frac{Q}{A} = \frac{4\times10^{-4}\ m^3/s}{(\pi/4)(0.02m)^2} = 1.27\ m/s$$

So that

$$\mathrm{Re} = \frac{\rho V D}{\mu} = \frac{VD}{\nu} = \frac{(1.27\ m/s)\times(0.02m)}{2.2\times10^{-4}\ m^2/s} = 115 < 2100$$

The flow is laminar with flowrate $Q = 4\times10^{-4}\ m^3/s$

Equation (8.12) in textbook reads:

$$Q = \frac{\pi(\Delta p - \gamma l \sin\theta)D^4}{128\mu l}$$

In this problem $\theta = -90$ (See textbook p411 for the sign convention for θ: θ is the angle between the pipe and the horizontal. $\theta > 0$ if the flow is uphill, while $\theta < 0$ if the flow is downhill.) So

$$Q = \frac{\pi(\Delta p + \gamma l)D^4}{128\mu l}, \text{ or } \Delta p = p_1 - p_2 = \frac{128\mu l Q}{\pi D^4} - \gamma l$$

Hence, with $\gamma = SG\gamma_{H_2O} = 0.87\times(9.81 kN/m^3) = 8.53 kN/m^3$ and

$$\mu = \rho\nu = \nu SG\rho_{H_2O} = (2.2\times10^{-4}\ m^2/s)(0.87)(1000\ kg/m^3) = 0.191\ Ns/m^2$$

The equation for Δp gives

$$\Delta p = \frac{128\times(0.191\ Ns/m^2)(4m)(4\times10^{-4}\ m^3/s)}{\pi(0.02m)^4} - (8.53\ kN/m^3)(4m)(10^3\ N/kN)$$

$$\Delta p = 4.37\times10^4\ N/m^2 = 43.7\ kN/m^2$$

From manometer considerations

$$p_1 + \gamma h_1 - \gamma_m h + \gamma h_2 = p_2$$

Where $\gamma_m = SG_m\gamma_{H_2O} = 1.3\times(9.81 kN/m^3) = 12.74 kN/m^3$ and

$$h_1 = h - h_2 + l \quad \text{or} \quad h_1 + h_2 = h + l$$

Thus

$$p_1 - p_2 = \Delta p = -\gamma\left(h_2 + h_1\right) + \gamma_m h = \left(\gamma_m - \gamma\right)h - \gamma l$$

Solve for h

$$h = \frac{\Delta p + \gamma l}{\left(\gamma_m - \gamma\right)} = \frac{43.7\times10^3\, N/m^2 + \left(8.53\times10^3\, N/m^3\right)\times\left(4m\right)}{\left(12.74 - 8.53\right)\times10^3\, N/m^3} = 18.5m$$

Discussion:

Case A: the flow is going down:
Suppose there is NO external pressure gradient, i.e., $\Delta p = 0$, the gravity of the fluid will create a flow downward with some flowrate. Let it be Q_g :

$$Q_g = \frac{\pi\left(\cancel{\Delta p} - \gamma l\sin\theta\right)D^4}{128\mu l} = \frac{\pi\left(-\gamma l\sin\left(-90^o\right)\right)D^4}{128\mu l} = \frac{\pi\gamma l D^4}{128\mu l}$$

In pipe flow the pressure difference is alwarys defined as $\Delta p = p_1 - p_2$ with p_1 upstram and p_2 downstram.

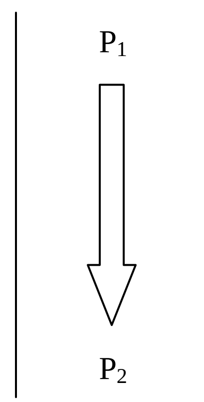
If $\Delta p = p_1 - p_2 > 0$, the flow is downward with $Q = \frac{\pi\left(\Delta p + \gamma l\right)D^4}{128\mu l} > Q_g$

If $\Delta p = p_1 - p_2 = 0$, the flow is downward with $Q = \frac{\pi\gamma l D^4}{128\mu l} = Q_g$

If $-\gamma l < \Delta p = p_1 - p_2 < 0$, the flow is downward with $0 < Q = \frac{\pi\left(\Delta p + \gamma l\right)D^4}{128\mu l} < Q_g$

If $-\gamma l = \Delta p = p_1 - p_2 < 0$, NO motion $0 = Q < Q_g$

If $\Delta p < -\gamma l$, the flow is upward with $Q = \frac{\pi\left(\Delta p + \gamma l\right)D^4}{128\mu l} < 0 < Q_g$



Case B: the flow is going up:
The pressure difference $\Delta p = p_1 - p_2$ should be at least large enough to overcome the gravity force.
i.e., $\Delta p = p_1 - p_2 > \gamma l$ so that $\Delta p - \gamma l > 0$

$$Q = \frac{\pi\left(\Delta p - \gamma l\sin\left(90^o\right)\right)D^4}{128\mu l} = \frac{\pi\left(\Delta p - \gamma l\right)D^4}{128\mu l} > 0$$