7.47

7.47 A thin layer of spherical particles rests on the bottom of a horizontal tube as shown in Fig. P7.47. When an incompressible fluid flows through the tube, it is observed that at some critical velocity the particles will rise and be transported along the tube. A model is to be used to determine this critical velocity. Assume the critical velocity, V_c to be a function of the pipe diameter, D, particle diameter, d, the fluid density, ρ, and viscosity, μ, the density of the particles, ρ_p, and the acceleration of gravity, g. **(a)** Determine the similarity requirements for the model, and the relationship between the critical velocity for model and prototype (the prediction equation). **(b)** For a length scale of $\frac{1}{2}$ and a fluid density scale of 1.0, what will be the critical velocity scale (assuming all similarity requirements are satisfied)?

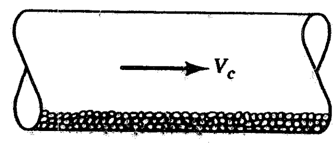

FIGURE P7.47

(a) $$V_c = f(D, d, \rho, \mu, \rho_p, g)$$

$V_c \doteq LT^{-1}$ $D \doteq L$ $d \doteq L$ $\rho \doteq FL^{-4}T^2$ $\mu \doteq FL^{-2}T$ $\rho_p \doteq FL^{-4}T^2$ $g \doteq LT^{-2}$

From the pi theorem, 7-3 = 4 pi terms required, and a dimensional analysis yields

$$\frac{\rho V_c D}{\mu} = \phi\left(\frac{d}{D}, \frac{\rho}{\rho_p}, \frac{g d^3 \rho^2}{\mu^2}\right)$$

Thus, the similarity requirements are

$$\frac{d_m}{D_m} = \frac{d}{D} \qquad \frac{\rho_m}{\rho_{pm}} = \frac{\rho}{\rho_p} \qquad \frac{g_m d_m^3 \rho_m^2}{\mu_m^2} = \frac{g d^3 \rho^2}{\mu^2}$$

The prediction equation is

$$\frac{\rho V_c D}{\mu} = \frac{\rho_m V_{cm} D_m}{\mu_m}$$

(b) If all similarity requirements are satisfied, the prediction equation indicates that

$$\frac{V_{cm}}{V_c} = \frac{\rho}{\rho_m}\frac{\mu_m}{\mu}\frac{D}{D_m} = (1.0)\left(\frac{\mu_m}{\mu}\right)(2) = 2\frac{\mu_m}{\mu} \qquad (1)$$

From the third similarity requirement (with $g = g_m$),

$$\frac{\mu_m}{\mu} = \sqrt{\left(\frac{d_m}{d}\right)^3\left(\frac{\rho_m}{\rho}\right)^2} = \sqrt{\left(\frac{1}{2}\right)^3(1.0)^2} = \sqrt{\frac{1}{8}}$$

Thus, from Eq. (1)

$$\frac{V_{cm}}{V_c} = 2\sqrt{\frac{1}{8}} = \underline{0.707}$$