7.13

7.13 Because of surface tension, it is possible, with care, to support an object heavier than water on the water surface as shown in Fig. P7.13. (See Video V1.5.) The maximum thickness, h, of a square of material that can be supported is assumed to be a function of the length of the side of the square, ℓ, the density of the material, ρ, the acceleration of gravity, g, and the surface tension of the liquid, σ. Develop a suitable set of dimensionless parameters for this problem.

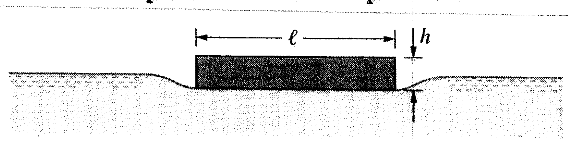

FIGURE P7.13

$$h = f(\ell, \rho, g, \sigma)$$

$h \doteq L \quad \ell \doteq L \quad \rho \doteq FL^{-4}T^{2} \quad g \doteq LT^{-2} \quad \sigma \doteq FL^{-1}$

From the pi theorem, $5-3=2$ pi terms required. Use ℓ, g, and ρ as repeating variables. Thus,

$$\Pi_1 = h\,\ell^a g^b \rho^c$$

and so that

$$(L)(L)^a(LT^{-2})^b(FL^{-4}T^2)^c \doteq F^0L^0T^0$$

$$c = 0 \qquad \text{(for } F)$$
$$1 + a + b - 4c = 0 \qquad \text{(for } L)$$
$$-2b + 2c = 0 \qquad \text{(for } T)$$

It follows that $a=-1$, $b=0$, $c=0$, and therefore

$$\Pi_1 = \frac{h}{\ell}$$

which is obviously dimensionless.

For Π_2:

$$\Pi_2 = \sigma\,\ell^a g^b \rho^c$$

$$(FL^{-1})(L)^a(LT^{-2})^b(FL^{-4}T^2)^c = F^0L^0T^0$$

$$1 + c = 0 \qquad \text{(for } F)$$
$$-1 + a + b - 4c = 0 \qquad \text{(for } L)$$
$$-2b + 2c = 0 \qquad \text{(for } T)$$

It follows that $a=-2$, $b=-1$, $c=-1$, and therefore

$$\Pi_2 = \frac{\sigma}{\ell^2 g\rho}$$

Check dimensions using MLT system:

$$\frac{\sigma}{\ell^2 g\rho} \doteq \frac{(MT^{-2})}{(L^2)(LT^{-2})(ML^{-3})} \doteq M^0L^0T^0 \quad \therefore \text{ ok}$$

Thus,

$$\underline{\underline{\frac{h}{\ell} = \phi\left(\frac{\sigma}{\ell^2 g\rho}\right)}}$$

Suppose the angle between the surface tension force and vertical direction (upward) is θ.

The vertical component of surface tension is in balance with the gavity. So we have

$$Mg = 4l\sigma\cos\theta$$

$$\left(\rho l^2 h\right)g = 4l\sigma\cos\theta$$

$$h = \frac{4l\sigma\cos\theta}{\left(\rho l^2\right)g}$$

$$\frac{h}{l} = \frac{\left(4\cos\theta\right)\sigma}{g\rho l^2}$$

$$\frac{h}{l} = f\left(\frac{\sigma}{g\rho l^2}\right)$$