6.80

6.80 An incompressible, viscous fluid is placed between horizontal, infinite, parallel plates as is shown in Fig. P6.80. The two plates move in opposite directions with constant velocities, U_1 and U_2, as shown. The pressure gradient in the x direction is zero and the only body force is due to the fluid weight. Use the Navier–Stokes equations to derive an expression for the velocity distribution between the plates. Assume laminar flow.

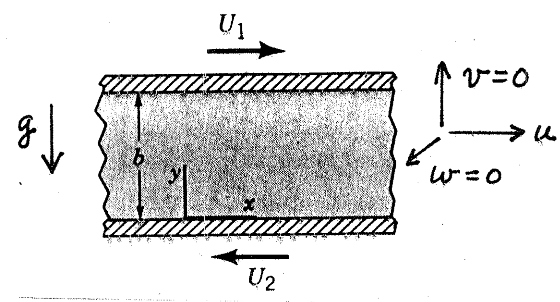

FIGURE P6.80

For the specified conditions, $v=0$, $w=0$, $\frac{\partial p}{\partial x}=0$, and $g_x=0$, so that the x-component of the Navier-Stokes equations (Eq. 6.127a) reduces to

$$\frac{d^2u}{dy^2}=0 \qquad (1)$$

Integration of Eq. (1) yields

$$u = C_1 y + C_2 \qquad (2)$$

For $y=0$, $u=-U_2$ and therefore from Eq. (2)

$$C_2 = -U_2$$

For $y=b$, $u=U_1$ so that

$$U_1 = C_1 b - U_2$$

or

$$C_1 = \frac{U_1+U_2}{b}$$

Thus,

$$u = \left(\frac{U_1+U_2}{b}\right) y - U_2$$