

6.38

6.38 The streamlines for an incompressible, inviscid, two-dimensional flow field are all concentric circles and the velocity varies directly with the distance from the common center of the streamlines; that is

$$v_\theta = Kr$$

where K is a constant. (a) For this rotational flow determine, if possible, the stream function. (b) Can the pressure difference between the origin and any other point be determined from the Bernoulli equation? Explain.

$$(a) \quad v_\theta = -\frac{\partial \psi}{\partial r} = Kr \quad (1)$$

Integrate Eq. (1) with respect to r to obtain

$$\int d\psi = -\int Kr dr$$

or

$$\psi = -\frac{Kr^2}{2} + f_1(\theta)$$

Since

$$v_r = \frac{1}{r} \frac{\partial \psi}{\partial \theta} = 0$$

it follows that ψ is not a function of θ and therefore

$$\psi = \underline{\underline{-\frac{Kr^2}{2} + C}}$$

where C is an arbitrary constant.

(b) The flow is rotational and therefore the Bernoulli equation cannot be applied between the origin and any point, since these points are not on the same streamline. No.

(Refer to discussion associated with derivation of Eq. 6.57.)