6.22 The stream function for an incompressible flow field is given by the equation

$$\psi = 3x^2y - y^3$$

where the stream function has the units of m²/s with x and y in meters. **(a)** Sketch the streamline(s) passing through the origin. **(b)** Determine the rate of flow across the straight path AB shown in Fig. P6.22.
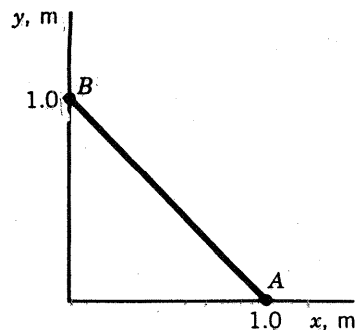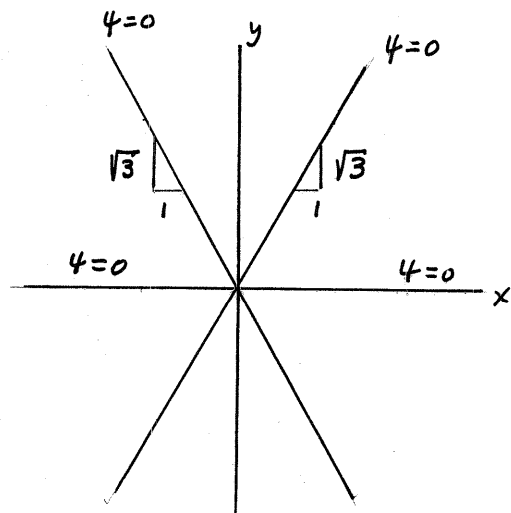

FIGURE P6.22

(a) Lines of constant ψ are streamlines. For $\psi = 3x^2y - y^3$ the streamline passing through the origin $(x=0, y=0)$ has a value $\psi = 0$. Thus, the equation for the streamlines through the origin is

$$0 = 3x^2y - y^3$$

or

$$y = \pm\sqrt{3}\,x$$

A sketch of these streamlines is shown in the figure.

(b)
$$q = \psi_B - \psi_A$$

At B $x=0$, $y=1m$ so that

$$\psi_B = 3(0)^2(1) - (1)^3 = -1 \ m^3/s \ (\text{per unit width})$$

At A $x=1m$, $y=0$ so that

$$\psi_A = 3(1)^2(0) - (0)^3 = 0$$

Thus,

$$q = \psi_B = \underline{-1 \ m^3/s \ (\text{per unit width})}$$

The negative sign indicates that the flow is from <u>right</u> to <u>left</u> as we look from A to B.