6.21

6.21 The radial velocity component in an incompressible, two-dimensional flow field ($v_z = 0$) is

$$v_r = 2r + 3r^2 \sin\theta$$

Determine the corresponding tangential velocity component, v_θ, required to satisfy conservation of mass.

$$\frac{1}{r}\frac{\partial(rv_r)}{\partial r} + \frac{1}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{\partial v_z}{\partial z} = 0 \qquad \text{(Eq. 6.35)}$$

Since $v_z = 0$,

$$\frac{\partial v_\theta}{\partial \theta} = -\frac{\partial(rv_r)}{\partial r} \qquad (1)$$

and with

$$rv_r = 2r^2 + 3r^3 \sin\theta$$

it follows that

$$\frac{\partial(rv_r)}{\partial r} = 4r + 9r^2 \sin\theta$$

Thus, Eq. (1) becomes

$$\frac{\partial v_\theta}{\partial \theta} = -(4r + 9r^2 \sin\theta) \qquad (2)$$

Equation (2) can be integrated with respect to θ to obtain

$$\int dv_\theta = -\int (4r + 9r^2 \sin\theta)\, d\theta + f(r)$$

or

$$v_\theta = \underline{\underline{-4r\theta - 9r^2\cos\theta + f(r)}}$$

where $f(r)$ is an undetermined function of r.