

6.20

6.20 In a two-dimensional, incompressible flow field, the x component of velocity is given by the equation $u = 2x$. (a) Determine the corresponding equation for the y component of velocity if $v = 0$ along the x axis. (b) For this flow field what is the magnitude of the average velocity of the fluid crossing the surface OA of Fig. P6.20? Assume that the velocities are in ft/s when x and y are in feet.

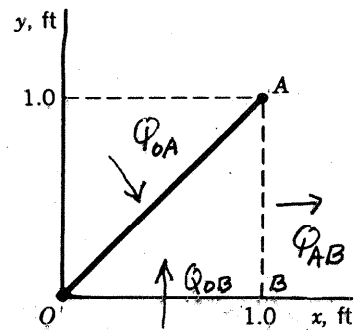


FIGURE P6.20

(a) To satisfy the continuity equation

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

Since $\frac{\partial u}{\partial x} = 2$

it follows that

$$\frac{\partial v}{\partial y} = -2 \quad (1)$$

Integration of Eq. (1) with respect to y yields

$$v = -2y + f(x)$$

If $v = 0$ along x -axis ($y = 0$) then $f(x) = 0$ so that

$$v = \underline{\underline{-2y}}$$

(b) To satisfy conservation of mass

$$Q_{OA} = Q_{AB} - Q_{OB} \quad (\text{see figure})$$

Along AB $u = 2(1) = 2 \frac{\text{ft}}{\text{s}}$ so that

$$Q_{AB} = u A_{AB} = (2 \text{ ft/s})(1 \text{ ft})(1 \text{ ft}) = 2 \frac{\text{ft}^3}{\text{s}}$$

Along OB $v = 0$ so that $Q_{OB} = 0$.

Thus,

$$Q_{OA} = Q_{AB} = 2 \frac{\text{ft}^3}{\text{s}}$$

and

$$V_{AV} = \frac{Q_{OA}}{\text{area}_{OA}} = \frac{2 \frac{\text{ft}^3}{\text{s}}}{\sqrt{2} \text{ ft}^2} = \underline{\underline{1.41 \frac{\text{ft}}{\text{s}}}}$$