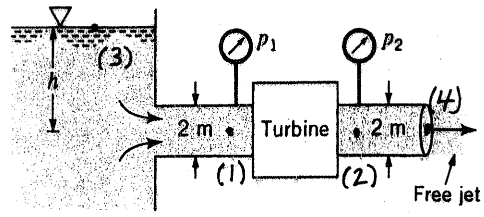
5.119

5.119 The turbine shown in Fig. P5.119 develops 2500 kW when the water flowrate is 20 m^3/s. The head loss across the turbine from (1) to (2) is negligible, but the head loss for the entire flow is 2.5 m. **(a)** Determine the pressure difference, $p_1 - p_2$, across the turbine. **(b)** Determine the elevation h.



■ FIGURE P5.119

(a) The energy equation across the turbine is

$$\frac{p_1}{\gamma} + z_1 + \frac{V_1^2}{2g} + h_s - h_{L_{1-2}} = \frac{p_2}{\gamma} + z_2 + \frac{V_2^2}{2g}, \text{ where } z_1 = z_2,\ h_{L_{1-2}} = 0,\ V_1 = V_2$$

Thus,

$$\frac{p_1}{\gamma} + h_s = \frac{p_2}{\gamma} \text{ or}$$

$$p_1 - p_2 = -\gamma h_s, \text{ where } h_s = \frac{\dot{W}_s}{\gamma Q}$$

Hence,

$$p_1 - p_2 = -\gamma\left(\frac{\dot{W}_s}{\gamma Q}\right) = -\frac{\dot{W}_s}{Q} = -\left(\frac{-2500\times10^3\ \frac{N\cdot m}{s}}{20\ \frac{m^3}{s}}\right) = 125\times10^3\ \frac{N}{m^2} = \underline{\underline{125\ kPa}}$$

(Note: $\dot{W}_s < 0$ because a turbine removes energy from the fluid.)

(b) Also, from (3) to (4)

$$\frac{p_3}{\gamma} + z_3 + \frac{V_3^2}{2g} + h_s - h_{L_{3-4}} = \frac{p_4}{\gamma} + z_4 + \frac{V_4^2}{2g}, \text{ where } p_3 = p_4 = 0,\ z_4 = 0,\ V_3 = 0,$$
and $z_3 = h$

Thus,

$$h + h_s - h_{L_{3-4}} = \frac{V_4^2}{2g}, \text{ or}$$

$$h = \frac{V_4^2}{2g} + h_{L_{3-4}} - h_s \qquad (1)$$

Also, $V_4 = \frac{Q}{A_4} = \frac{20\ \frac{m^3}{s}}{\left(\frac{\pi}{4}(2m)^2\right)} = 6.37\ \frac{m}{s}$, $h_{L_{3-4}} = 2.5m$, and

$$h_s = \frac{\dot{W}_s}{\gamma Q} = \frac{-2500\times10^3\ \frac{N\cdot m}{s}}{\left(9.83\times10^3\ \frac{N}{m^3}\right)\left(20\ \frac{m^3}{s}\right)} = -12.76m$$

Therefore, from Eq. (1)

$$h = \frac{(6.37\ \frac{m}{s})^2}{2(9.81\ \frac{m}{s^2})} + 2.5m - (-12.76m) = \underline{\underline{17.3\ m}}$$