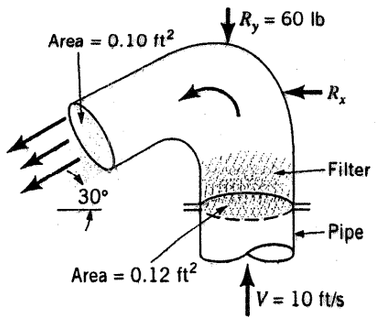
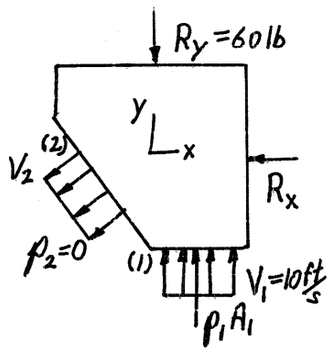
5.103

5.103 Water flows steadily in a pipe and exits as a free jet through an end cap that contains a filter as shown in Fig. P5.103. The flow is in a horizontal plane. The axial component, R_y, of the anchoring force needed to keep the end cap stationary is 60 lb. Determine the head loss for the flow through the end cap.



■ **FIGURE P5.103**



The y-component of the momentum equation, $\int_{cs} \rho \vec{V}\cdot\hat{n}\, dA = \sum F_y$, for the control volume shown is

(1) $V_1 \rho(-V_1)A_1 + (-V_2 \sin 30°)\rho V_2 A_2 = p_1 A_1 - R_y$

where $V_1 = 10$ ft/s and

$$V_2 = \frac{A_1}{A_2} V_1 = \left(\frac{0.12\ \text{ft}^2}{0.10\ \text{ft}^2}\right)(10\ \text{ft/s}) = 12\ \text{ft/s}$$

Thus, since $\rho A_1 V_1 = \rho A_2 V_2$, Eq. (1) gives

$$p_1 A_1 = R_y - \rho V_1^2 A_1 - \rho V_2^2 \sin 30° A_2 = R_y - \rho A_1 V_1 [V_1 + V_2 \sin 30°]$$

$$= 60\ \text{lb} - \left(1.94\ \frac{\text{slugs}}{\text{ft}^3}\right)(0.12\ \text{ft}^2)(10\ \tfrac{\text{ft}}{\text{s}})\left[10\ \tfrac{\text{ft}}{\text{s}} + 12\ \tfrac{\text{ft}}{\text{s}} \sin 30°\right] = 22.8\ \text{lb}$$

Hence,

$$p_1 = 22.8\ \text{lb}/A_1 = 22.8\ \text{lb}/(0.12\ \text{ft}^2) = 190\ \text{lb/ft}^2$$

From the energy equation for this flow,

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} - h_L = \frac{V_2^2}{2g}, \text{ or}$$

$$h_L = \frac{p_1}{\gamma} + \frac{V_1^2 - V_2^2}{2g} = \frac{190\ \text{lb/ft}^2}{62.4\ \text{lb/ft}^3} + \frac{(10\ \text{ft/s})^2 - (12\ \text{ft/s})^2}{2(32.2\ \text{ft/s}^2)} = \underline{\underline{2.36\ \text{ft}}}$$