5.90

5.90 Oil ($SG = 0.9$) flows downward through a vertical pipe contraction as shown in Fig. P5.90. If the mercury manometer reading, $h$, is 100 mm, determine the volume flowrate for frictionless flow. Is the actual flowrate more or less than the frictionless value? Explain.

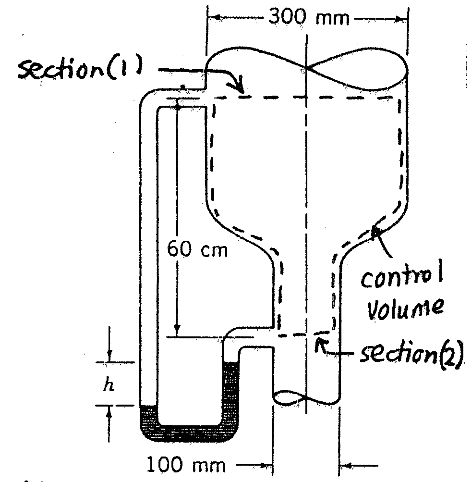


FIGURE P5.90

The volume flowrate may be obtained with

$$Q = A_1 V_1 = A_2 V_2 = \frac{\pi D_1^2}{4} V_1 = \frac{\pi D_2^2}{4} V_2 \qquad (1)$$

To determine either $V_1$ or $V_2$ we apply the energy equation (Eq. 5.82) to the flow between sections (1) and (2). Thus,

$$\frac{p_2}{\rho} + \frac{V_2^2}{2} + g z_2 = \frac{p_1}{\rho} + \frac{V_1^2}{2} + g z_1 + \underbrace{\cancel{w_{shaft\ net\ in}}}_{0} - \underbrace{\cancel{loss}}_{neglect} \qquad (2)$$

Combining Eqs. 1 and 2 we obtain

$$\frac{V_2^2}{2}\left[1 - \left(\frac{D_2}{D_1}\right)^4\right] = \frac{p_1 - p_2}{\rho} + g(z_1 - z_2) \qquad (3)$$

To determine $\frac{p_1 - p_2}{\rho}$ we use the manometer equation from section 2.6 to obtain

$$\frac{p_1 - p_2}{\rho} = gh\left(\frac{SG_{Hg}}{SG_{oil}} - 1\right) - g(z_1 - z_2) \qquad (4)$$

Combining Eqs. 3 and 4 we get

$$V_2 = \sqrt{\frac{2gh\left(\frac{SG_{Hg}}{SG_{oil}} - 1\right)}{1 - \left(\frac{D_2}{D_1}\right)^4}}$$

or

$$V_2 = \sqrt{\frac{(2)(9.81\ \frac{m}{s^2})(0.1\ m)\left(\frac{13.6}{0.9} - 1\right)}{1 - \left(\frac{100\ mm}{300\ mm}\right)^4}} = 5.29\ \frac{m}{s}$$

and from Eq. 1 we have

$$Q = \frac{\pi (0.1 m)^2}{4}\left(5.29\ \frac{m}{s}\right) = \underline{\underline{0.042\ \frac{m^3}{s}}}$$

Actual flowrate would be less than the frictionless value because the loss would be greater than the zero amount used above.