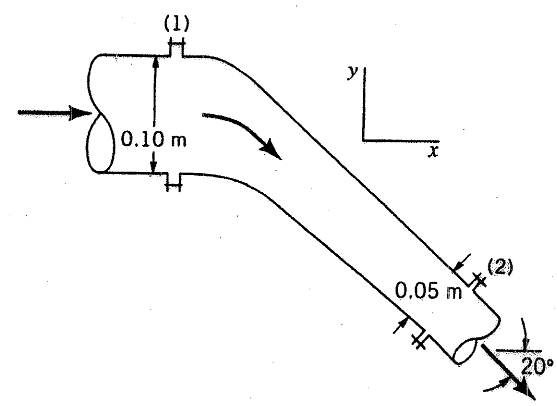
5.31

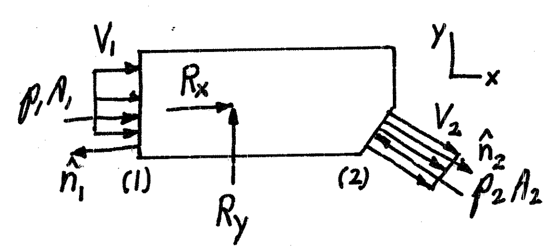
5.31 Water flows through the 20° reducing bend shown in Fig. P5.31 at a rate of 0.025 m^3/s. The flow is frictionless, gravitational effects are negligible, and the pressure at section (1) is 150 kPa. Determine the x and y components of force required to hold the bend in place.



■ FIGURE P5.31

For the control volume shown the x-component of the momentum equation is

$$\int_{cs} u\rho\vec{V}\cdot\hat{n}\,dA = \Sigma F_x, \text{ or}$$



$$V_1\rho(-V_1)A_1 + (V_2\cos 20^\circ)\rho V_2 A_2 = R_x + p_1A_1 - p_2A_2\cos 20^\circ$$

or

(1) $R_x = p_2A_2\cos 20^\circ - p_1A_1 + (V_2\cos 20^\circ - V_1)\dot{m}$, where $\dot{m} = \dot{m}_1 = \dot{m}_2 = \rho Q = \rho AV$

Also,

$$V_1 = Q/A = (0.025\tfrac{m^3}{s})/(\tfrac{\pi}{4}(0.10m)^2) = 3.18\tfrac{m}{s}$$

and

$$V_2 = Q/A_2 = (0.025\tfrac{m^3}{s})/(\tfrac{\pi}{4}(0.05m)^2) = 12.7\tfrac{m}{s}$$

In addition, from the Bernoulli equation,

$$p_1 + \tfrac{1}{2}\rho V_1^2 = p_2 + \tfrac{1}{2}\rho V_2^2, \text{ or}$$

$$p_2 = p_1 + \tfrac{1}{2}\rho(V_1^2 - V_2^2) = 150\,kPa + \tfrac{1}{2}(999\tfrac{kg}{m^3})[(3.18\tfrac{m}{s})^2 - (12.7\tfrac{m}{s})^2]$$

$$= 150\times10^3\tfrac{N}{m^2} - 75.5\times10^3\tfrac{kg\cdot m}{s^2}/m^2 = 74.5\,kPa$$

Thus, from Eq.(1),

$$R_x = 74.5\times10^3\tfrac{N}{m^2}(\tfrac{\pi}{4}(0.05m)^2)\cos 20^\circ - 150\times10^3\tfrac{N}{m^2}(\tfrac{\pi}{4}(0.1\,m)^2)$$

$$+[(12.7\tfrac{m}{s})\cos 20^\circ - 3.18\tfrac{m}{s}](999\tfrac{kg}{m^3})(0.025\tfrac{m^3}{s}) = \underline{\underline{-822\,N}}$$

Similarly, in the y-direction $\int_{cs} v\rho\vec{V}\cdot\hat{n}\,dA = \Sigma F_y$, or

$$(-V_2\sin 20^\circ)\rho V_2A_2 = p_2A_2\sin 20^\circ + R_y$$

or

(2) $R_y = -V_2\sin 20^\circ\,\dot{m} - p_2A_2\sin 20^\circ$

$$= -(12.7\tfrac{m}{s})\sin 20^\circ(999\tfrac{kg}{m^3})(0.025\tfrac{m^3}{s}) - 74.5\times10^3\tfrac{N}{m^2}(\tfrac{\pi}{4}(0.05m)^2)\sin 20^\circ$$

$$= \underline{\underline{-156\,N}}$$