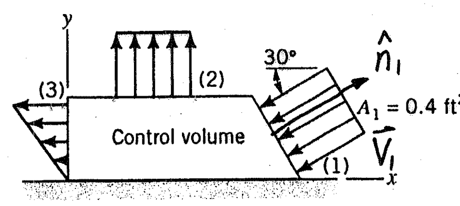
5.17

**5.17** Water flows steadily through the control volume shown in Fig. P5.17. The volumetric flowrate across section (3) is 2 ft³/s and the mass flowrate across section (2) is 3 slugs/s.




■ FIGURE P5.17

(a) $\int_{(1)} \gamma \vec{V}\cdot\hat{n}\, dA$ = -weight flowrate across area (1) $= -g\dot{m}_1$ (1)

(Note: $\int_{(1)} \gamma \vec{V}\cdot\hat{n}\, dA < 0$ since $\vec{V}\cdot\hat{n} < 0$ for the inflow area (1))

By conservation of mass, for steady flow,

$$\dot{m}_1 = \dot{m}_2 + \dot{m}_3 = \dot{m}_2 + \rho_3 Q_3 = 3\,slugs/s + (1.94\,slugs/ft^3)(2\,ft^3/s)$$

or

$$\dot{m}_1 = 6.88\,slugs/s \qquad (2)$$

Thus, from Eq. (1),

$$\int_{(1)} \gamma \vec{V}\cdot\hat{n}\, dA = (-32.2\,ft/s^2)(6.88\,slugs/s) = -222\,(slug\cdot ft/s^2)/s = \underline{\underline{-222\,lb/s}}$$

(b) $\int_{(1)} \vec{V}\rho\vec{V}\cdot\hat{n}\, dA$ = momentum flux across area (1)

On (1), $\hat{n}_1 = +\cos 30°\,\hat{i} + \sin 30°\,\hat{j}$ and

$\vec{V}_1 = V_1(-\cos 30°\,\hat{i} - \sin 30°\,\hat{j}) = -V_1\hat{n}_1$

where $\rho A_1 V_1 = \dot{m}_1 = 6.88\,\frac{slugs}{s}$ (from Eq. (2))

Thus,

$$V_1 = \frac{\dot{m}_1}{\rho A_1} = \frac{6.88\,\frac{slugs}{s}}{(1.94\,\frac{slugs}{ft^3})(0.4\,ft^2)} = 8.87\,\frac{ft}{s}$$

Hence,

$$\int_{(1)} \vec{V}_1\rho\vec{V}_1\cdot\hat{n}_1\, dA = \vec{V}_1\rho(-V_1\hat{n}_1)\cdot\hat{n}_1 A_1 = -\rho V_1 A_1 \vec{V}_1 = -\dot{m}_1\vec{V}_1 = +\dot{m}_1 V_1\hat{n}_1$$

$$= (6.88\,\frac{slugs}{s})(8.87\,\frac{ft}{s})(\cos 30°\,\hat{i} + \sin 30°\,\hat{j})$$

$$= (52.8\,\hat{i} + 30.5\,\hat{j})\,\frac{slug\cdot ft}{s^2}$$

$$= \underline{\underline{52.8\,\hat{i} + 30.5\,\hat{j}\ lb}}$$