

4.60

4.60 Water flows through the 2-m-wide rectangular channel shown in Fig. P4.60 with a uniform velocity of 3 m/s. (a) Directly integrate Eq. 4.16 with $b = 1$ to determine the mass flowrate (kg/s) across section CD of the control volume. (b) Repeat part (a) with $b = 1/\rho$, where ρ is the density. Explain the physical interpretation of the answer to part (b).

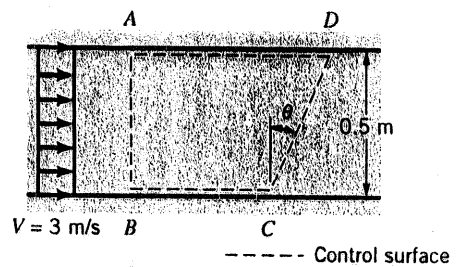


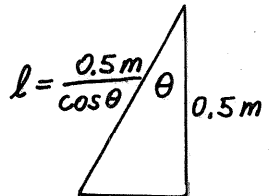
FIGURE P4.60

$$a) \dot{B}_{out} = \int_{CS_{out}} \rho b \vec{V} \cdot \hat{n} dA \quad (1)$$

With $b = 1$ and $\vec{V} \cdot \hat{n} = V \cos \theta$ this becomes

$$\dot{B}_{out} = \int_{CD} \rho V \cos \theta dA = \rho V \cos \theta \int_{CD} dA$$

$$= \rho V \cos \theta A_{CD}, \quad \text{where } A_{CD} = l (2m) \\ = \left(\frac{0.5m}{\cos \theta} \right) (2m) \\ = \left(\frac{1}{\cos \theta} \right) m^2$$



Thus, with $V = 3 \text{ m/s}$,

$$\dot{B}_{out} = \left(3 \frac{m}{s} \right) \cos \theta \left(\frac{1}{\cos \theta} \right) m^2 (999 \frac{kg}{m^3}) = \underline{\underline{3000 \frac{kg}{s}}}$$

b) With $b = 1/\rho$ Eq. (1) becomes

$$\dot{B}_{out} = \int_{CD} \vec{V} \cdot \hat{n} dA = \int_{CD} V \cos \theta dA = V \cos \theta A_{CD} \\ = \left(3 \frac{m}{s} \right) \cos \theta \left(\frac{1}{\cos \theta} \right) m^2 = \underline{\underline{3.00 \frac{m^3}{s}}}$$

With $b = 1/\rho = \frac{1}{\left(\frac{mass}{vol} \right)} = \frac{vol}{mass}$ it follows that "B = volume" (i.e., $b = \frac{B}{mass}$) so that $\int \vec{V} \cdot \hat{n} dA = \dot{B}_{out}$ represents the volume flowrate (m^3/s) from the control volume.