4.55

4.55 A layer of oil flows down a vertical plate as shown in Fig. P4.55 with a velocity of $\mathbf{V} = (V_0/h^2)(2hx - x^2)\hat{\mathbf{j}}$ where V_0 and h are constants. (a) Show that the fluid sticks to the plate and that the shear stress at the edge of the layer ($x = h$) is zero. (b) Determine the flowrate across surface AB. Assume the width of the plate is b. (Note: The velocity profile for laminar flow in a pipe has a similar shape. See Video V6.6.)

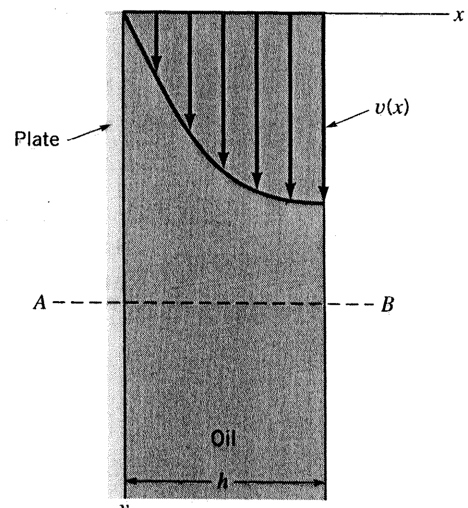

FIGURE P4.55

a) $v = \frac{V_0}{h^2}(2hx - x^2)$

Thus,

$$v\Big|_{x=0} = \frac{V_0}{h^2}(0-0) = 0 \quad \text{and}$$

$$\tau\Big|_{x=h} = \mu \frac{dv}{dx}\Big|_{x=h} = \mu \frac{V_0}{h^2}\left[2h - 2x\right]_{x=h} = 0$$

Hence, the fluid sticks to the plate and there is no shear stress at the free surface.

b) $$Q_{AB} = \int v\, dA = \int_{x=0}^{x=h} v\, b\, dx = \int_0^h \frac{V_0}{h^2}(2hx - x^2)\, b\, dx$$

or

$$Q_{AB} = \frac{V_0 b}{h^2}\left[hx^2 - \frac{1}{3}x^3\right]_0^h = \underline{\underline{\frac{2}{3}V_0 h b}}$$