4.51

4.51 In the region just downstream of a sluice gate, the water may develop a reverse flow region as is indicated in Fig. P4.51 and Video V10.5. The velocity profile is assumed to consist of two uniform regions, one with velocity $V_a = 10$ fps and the other with $V_b = 3$ fps. Determine the net flowrate of water across the portion of the control surface at section (2) if the channel is 20 ft wide.

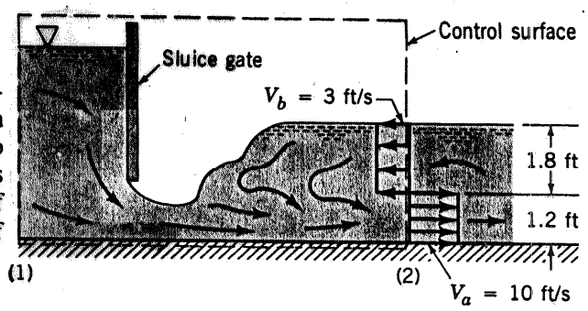

FIGURE P4.51

$$Q = V_a A_a - V_b A_b = (10\,\tfrac{ft}{s})(1.2ft)(20ft) - (3\,\tfrac{ft}{s})(1.8ft)(20ft)$$
$$= 132\,\tfrac{ft^3}{s}$$

4.52

4.52 At time $t = 0$ the valve on an initially empty (perfect vacuum, $\rho = 0$) tank is opened and air rushes in. If the tank has a volume of $\forall_0$ and the density of air within the tank increases as $\rho = \rho_\infty(1 - e^{-bt})$, where b is a constant, determine the time rate of change of mass within the tank.

For $t \geq 0$, $\rho = \rho_0[1 - e^{-bt}]$ so that M = mass of air in tank
$= \rho \forall_0 = \rho_0 \forall_0 [1 - e^{-bt}]$

Thus, $\frac{dM}{dt} = \rho_0 \forall_0 b\, e^{-bt}$