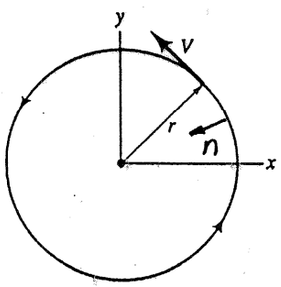
3.12

3.12 Water in a container and air in a tornado flow in horizontal circular streamlines of radius r and speed V as shown in Video V3.2 and Fig. P3.12. Determine the radial pressure gradient, $\partial p/\partial r$, needed for the following situations: (a) The fluid is water with $r = 3$ in. and $V = 0.8$ ft/s. (b) The fluid is air with $r = 300$ ft and $V = 200$ mph.



■ FIGURE P3.12

For curved streamlines,

$$-\frac{dp}{dn} = \frac{\rho V^2}{R} + \gamma \frac{dz}{dn}, \quad \text{or with } \frac{dz}{dn} = 0 \text{ (horizontal streamlines)}, \ R = r,$$

and $\frac{d}{dn} = -\frac{d}{dr}$ this becomes

$$\frac{dp}{dr} = \frac{\rho V^2}{r}$$

a) With $r = \frac{3}{12}$ ft and $V = 0.8 \frac{ft}{s}$ and water $(\rho = 1.94 \frac{slugs}{ft^3})$,

$$\frac{dp}{dr} = \frac{1.94 \frac{slugs}{ft^3} (0.8 \frac{ft}{s})^2}{(\frac{3}{12} ft)} = 4.97 \frac{slugs}{ft^2 \cdot s^2} = \underline{\underline{4.97 \frac{lb}{ft^3}}}$$

(b) With $r = 300$ ft and $V = 200 \text{ mph} \left(\frac{88 \frac{ft}{s}}{60 \text{ mph}}\right) = 293 \frac{ft}{s}$

and air $(\rho = 0.00238 \frac{slugs}{ft^3})$,

$$\frac{dp}{dr} = \frac{0.00238 \frac{slugs}{ft^3} (293 \frac{ft}{s})^2}{300 \text{ ft}} = 0.681 \frac{slugs}{ft^2 \cdot s^2} = \underline{\underline{0.681 \frac{lb}{ft^3}}}$$