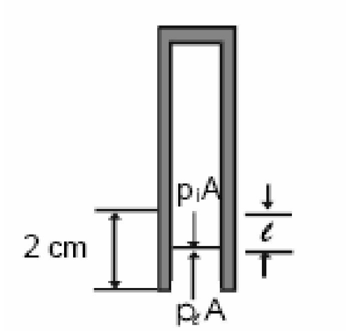
3.5 Problem statement

A glass tube 10cm long and 0.5mm internal diameter has one end closed. The tube is inserted into water to a depth of 2cm, as shown. In the process of inserting the tube, the air is trapped inside and undergoes a constant temperature compression. The atmospheric pressure is 100kPa, and the water density is 1000kg/m^3. Find the location of the water level in the tube including the effects of surface tension.

Find

Location of water line in tube



Solution

(a) Assume water wets the glass

Equate forces acting at the liquid surface inside the glass tube

$$\sum F_z = 0$$

$$-p_i A + p_l A + \sigma \pi d = 0 \qquad (1)$$

Where p_i is the pressure inside the tube and p_l is the pressure in water at depth l. Also

$$p_i \forall_i = p_{atm} \forall_{tube}$$

$$p_i = p_{atm}\left(\forall_{tube} / \forall_i\right)$$

$$= p_{atm}\left(0.10 A_{tube} / \left((.08 + l)(A_{tube})\right)\right)$$

$$p_i = p_{atm}\left(0.10 / (.08 + l)\right) \qquad (2)$$

$$p_l = p_{atm} + \gamma l \qquad (3)$$

Solve for l with Eqs. (1), (2), and (3)

$$-\left(p_{atm}\frac{0.10}{.08+l}\right)\left(\frac{1}{4}\pi d^2\right) + \left(p_{atm} + \gamma l\right)\left(\frac{1}{4}\pi d^2\right) + \sigma \pi d = 0$$

$$-\left(p_{atm}\frac{0.10}{.08+l}\right)\frac{d}{4} + \left(p_{atm} + \gamma l\right)\frac{d}{4} + \sigma = 0$$

$$-\left(10^5\frac{0.10}{.08+l}\right)\frac{0.0005}{4} + \left(10^5 + 1000 \times 9.8 \times l\right)\frac{0.0005}{4} + 0.073 = 0$$

$$l = 0.0192334m = \boxed{1.92cm}$$

(b) Assume there is NO effect of surface tension. Simply neglect the surface tension term in the above equations and solve for l

$$l = 0.0198063m = \boxed{1.98cm}$$

(c) Assume the air inside the tube is incompressible. The top end of the tube must be open to the atmospheric pressure in order for water to get into the tube. Consider ONLY the effects of surface tension, which make the water surface inside the tube goes above the water surface outside the tube:

$$l = \frac{4\sigma}{\gamma d} = \frac{4 \times 0.073}{1000 \times 9.8 \times 0.0005} = 0.05959m = \boxed{5.959cm}$$