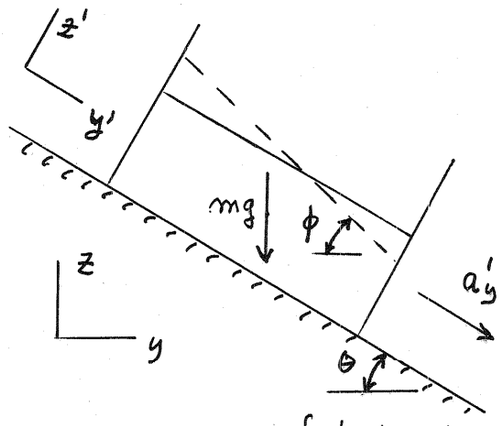
2.95

2.95 If the tank of Problem 2.94 slides down a frictionless plane that is inclined at 30° with the horizontal, determine the angle the free surface makes with the horizontal.



m ~ mass of tank and gasoline

From Newton's 2nd law,

$$\Sigma F_{y'} = m a_{y'}$$

Since the only force in the y'-direction is the component of weight $(mg)\sin\theta$,

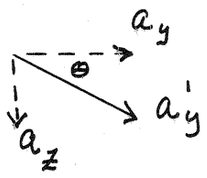
$$(mg)\sin\theta = m a_{y'}$$

so that

$$a_{y'} = g\sin\theta$$

and therefore

$$a_y = a_{y'}\cos\theta \qquad a_z = -a_{y'}\sin\theta$$



Also,

$$\frac{dz}{dy} = -\frac{a_y}{g + a_z} \qquad \text{(Eq. 2.28)}$$

$$= -\frac{a_{y'}\cos\theta}{g - a_{y'}\sin\theta} = -\frac{g\sin\theta\cos\theta}{g - g\sin\theta\cos\theta}$$

$$= -\frac{\frac{1}{2}\sin 2\theta}{1 - \frac{1}{2}\sin 2\theta}$$

and for $\theta = 30°$

$$\frac{dz}{dy} = -\frac{\frac{1}{2}\sin 60°}{1 - \frac{1}{2}\sin 60°} = -0.764$$

Thus, $\tan\phi = 0.764$ (see figure)

and

$$\underline{\underline{\phi = 37.4°}}$$