Chapter 8 Flow in Conduits

Entrance and developed flows

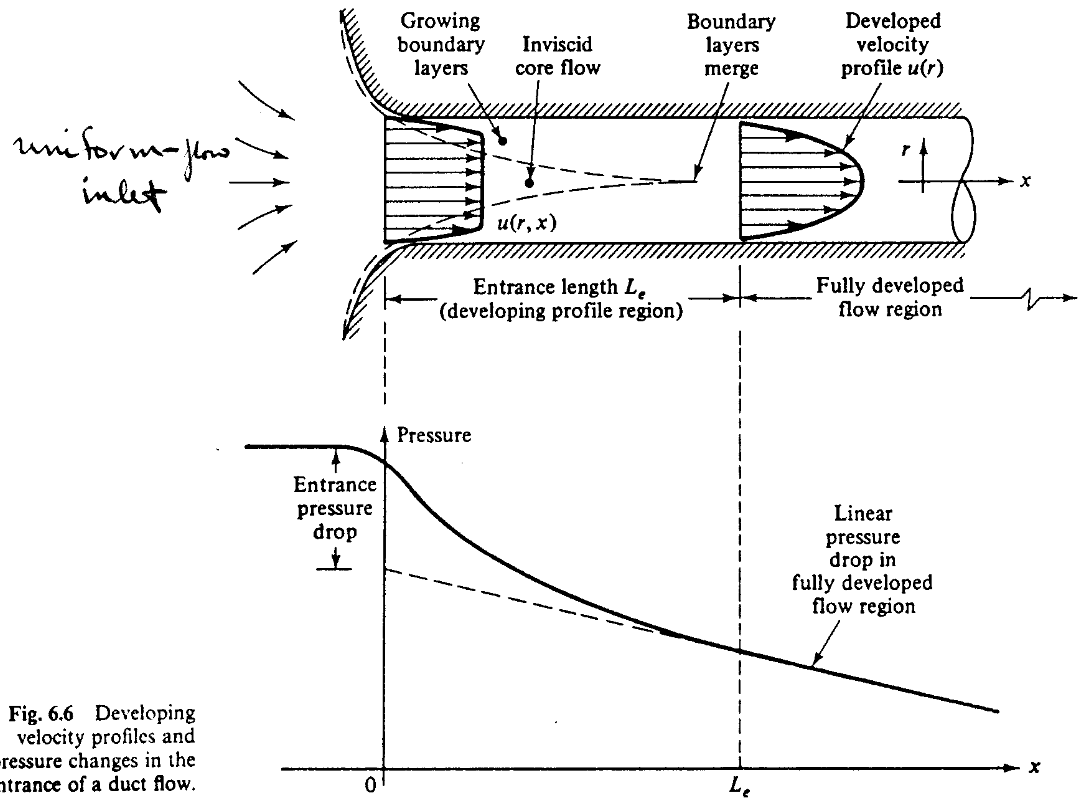


Fig. 6.6 Developing velocity profiles and pressure changes in the entrance of a duct flow.

$$Le = f(D, V, \rho, \mu)$$

Π_i theorem $\Rightarrow$ $Le/D = f(Re)$

Laminar flow: $Re_{crit} \sim 2000$, i.e., for $Re < Re_{crit}$ laminar
$Re > Re_{crit}$ turbulent

$Le/D = .06Re$ from experiments

$Le_{max} = .06Re_{crit}D \sim 138D$

maximum Le for laminar flow

Turbulent flow:

$$\frac{Le}{D} \sim 4.4\,Re^{1/6}$$

from experiment

Re	Le/D
4000	18
10^4	20
10^5	30
10^6	44
10^7	65
10^8	95

i.e., relatively shorter than for laminar flow

Laminar vs. Turbulent Flow

Hagen 1839 noted difference in $\Delta p = \Delta p(V)$ but could not explain two regimes

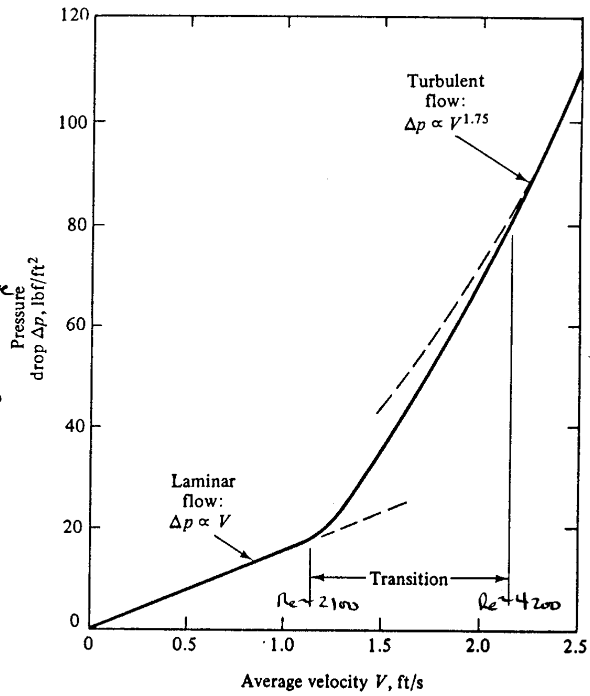
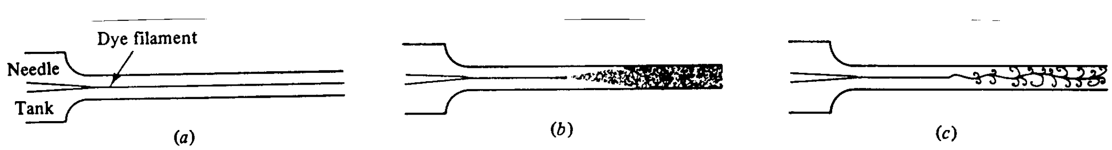


Fig. 6.4 Experimental evidence of transition for water flow in a $\frac{1}{2}$-in smooth pipe 10 ft long.



laminar turbulent spark photo

Reynolds 1883 showed difference depends on $Re = \dfrac{VD}{\nu}$

8.1 Shear-Stress Distribution Across a Pipe Section

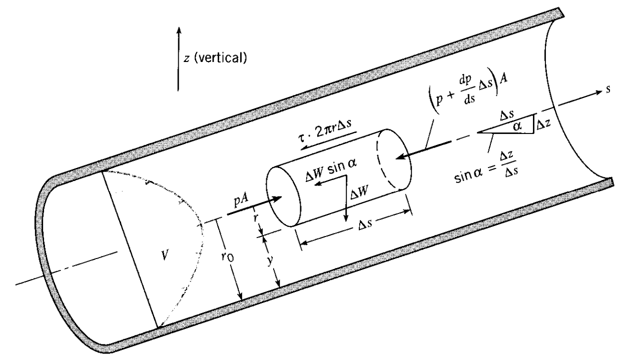


FIGURE 10.1
Variation of shear stress in a pipe.

Continuity: $Q_1 = Q_2 = \text{constant}$
i.e., $V_1 = V_2$ since $A_1 = A_2$

Momentum:
$$\sum F_s = \sum \rho u(\underline{V}\cdot\underline{A})$$
$$= \rho V_1(-V_1A_1) - \rho V_2(V_2A_2)$$
$$= \rho Q(V_2 - V_1) = 0$$

$$pA - \left(p + \frac{dp}{ds}ds\right)A - \Delta W \sin\alpha - \tau(2\pi r)ds = 0$$

$$\Delta W = \gamma A ds \qquad \sin\alpha = \frac{dz}{ds}$$

$$-\frac{dp}{ds}dsA - \gamma A ds\frac{dz}{ds} - \tau(2\pi r)ds = 0$$

$\div$ Ads
$$\tau = \frac{r}{2}\left[-\frac{d}{ds}(p + \gamma z)\right]$$

τ varies linearly from 0.0 at r = 0 (centerline) to τ_{max} (= τ_w) at r = R (wall). Valid for laminar and turbulent flow.

8.2 Laminar Flow in Pipes

$$\tau = \mu \frac{dV}{dy} = -\mu \frac{dV}{dr} = \frac{r}{2}\left[-\frac{d}{ds}(p+\gamma z)\right]$$

$$y = \text{wall coordinate} = r_o - r \Rightarrow \frac{dV}{dr} = \frac{dV}{dy}\frac{dy}{dr} = -\frac{dV}{dy}$$

$$\frac{dV}{dr} = -\frac{r}{2\mu}\left[-\frac{d}{ds}(p+\gamma z)\right]$$

$$V = -\frac{r^2}{4\mu}\left[-\frac{d}{ds}(p+\gamma z)\right] + C$$

$$\underbrace{V(r_o) = 0}_{\text{no slip condition}} \Rightarrow C = \frac{r_o^2}{4\mu}\left[-\frac{d}{ds}(p+\gamma z)\right]$$

$$V(r) = \frac{r_o^2 - r^2}{4\mu}\left[-\frac{d}{ds}(p+\gamma z)\right]$$

Exact solution to Navier-Stokes equations for laminar flow in circular pipe

$$Q = \int \underline{V} \cdot d\underline{A}$$

$$= \int_0^{r_o} V(r)\underbrace{2\pi r dr}$$

$$V_{max} = \frac{r_o^2}{4\mu}\left[-\frac{d}{ds}(p+\gamma z)\right]$$

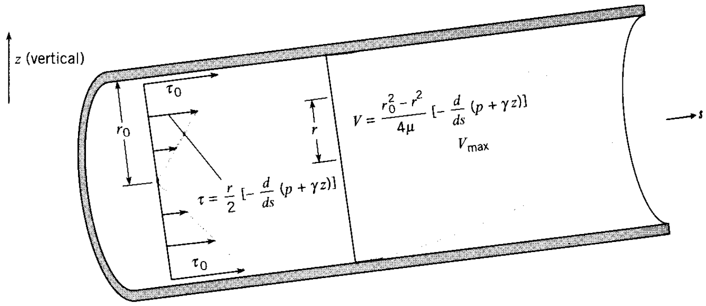
$$dA = r dr d\theta = r dr(2\pi)$$

$$Q = \frac{\pi r_o^4}{8\mu}\left[-\frac{d}{ds}(p+\gamma z)\right]$$

$$\overline{V} = \frac{Q}{A} = \frac{r_o^2}{8\mu}\left[-\frac{d}{ds}(p+\gamma z)\right]$$

$$\overline{V} = \frac{V_{max}}{2}$$

FIGURE 10.2
Distribution of shear stress and velocity for laminar flow in a pipe.



energy equation:

$$\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+z_1=\frac{p_2}{\gamma}+\frac{V_2^2}{2g}+z_2+h_L$$

$$\Delta h=\left(\frac{p_2}{\gamma}+z_2\right)-\left(\frac{p_1}{\gamma}+z_1\right)$$

$$h_L=\frac{p_1-p_2}{\gamma}+(z_1-z_2)=-\Delta h$$

$$h_L=\frac{L}{\gamma}\left[-\frac{d}{ds}(p+\gamma z)\right]$$

L = length of pipe = ds

$$h_L=L\left[-\frac{d}{ds}\left(\frac{p}{\gamma}+z\right)\right]$$

$$=L\left(-\frac{dh}{ds}\right)$$

$$=\frac{L}{\gamma}\left[\frac{8\mu\overline{V}}{r_o^2}\right]=-\Delta h\alpha\overline{V}$$

or $$h_f=h_L=\frac{32\mu L\overline{V}}{\gamma D^2}$$

h_f = head loss due to friction

exact solution

$$\underbrace{\text{friction factor } f = \frac{8\tau_w}{\rho \overline{V}^2}}_{\text{friction coefficient for pipe flow}} \qquad \underbrace{C_f = \frac{\tau_w}{\frac{1}{2}\rho V^2}}_{\text{boundary layer flow}}$$

$$\tau_w = \frac{r_o}{2}\left[-\frac{d}{ds}(p + \gamma z)\right]$$

$$= \frac{r_o}{2}\left[\frac{8\mu \overline{V}}{r_o^2}\right]$$

$$\tau_w = \frac{4\mu \overline{V}}{r_o}$$

$$f = \frac{32\mu}{\rho r_o \overline{V}} = \frac{64\mu}{\rho \overline{V} D} = \frac{64}{Re} \qquad \text{exact solution}$$

$$Re = \frac{\overline{V} D}{\nu} \qquad \nu = \frac{\mu}{\rho}$$

8.3 Criterion for Laminar or Turbulent Flow in a Pipe

$Re_{crit} \sim 2000$ flow becomes unstable
$Re_{trans} \sim 3000$ flow becomes turbulent
$Re = \overline{V}D/\nu$

8.4 Turbulent Flow in Pipes

Continuity and momentum:

$$\tau(r = r_o) = \tau_o = \frac{r_o}{2}\left[-\frac{d}{ds}(p + \gamma z)\right]$$

Energy: $$h_f = \frac{L}{\gamma}\left[-\frac{d}{ds}(p + \gamma z)\right]$$

Combining: $h_f = \frac{L}{\gamma} \cdot \frac{2\tau_o}{r_o}$ define $f = \dfrac{\tau_o}{\frac{1}{8}\rho\overline{V}^2}$ = friction factor

$$h_f = \frac{L}{\rho g} \cdot \frac{2}{r_o} \cdot \frac{1}{8}\rho\overline{V}^2 f$$

$$h_f = f \cdot \frac{L}{D} \cdot \frac{\overline{V}^2}{2g}$$ Darcy – Weisbach Equation

f = f(Re, k/D) = still must be determined!

$Re = \dfrac{\overline{V}D}{\nu}$ k = roughness

Velocity Distribution and Resistance in Smooth Pipes

As with turbulent boundary layers, mean-velocity follows three layer concept:

1. laminar sub-layer (viscous shear dominates)

$$u^+ = y^+ \qquad 0 < y^+ < 5 \qquad y^+ = \frac{yu^*}{\gamma} \qquad y = r_o - r$$

$$u^* = \sqrt{\frac{\tau_o}{\rho}}$$

2. overlap layer (viscous and turbulent shear important)

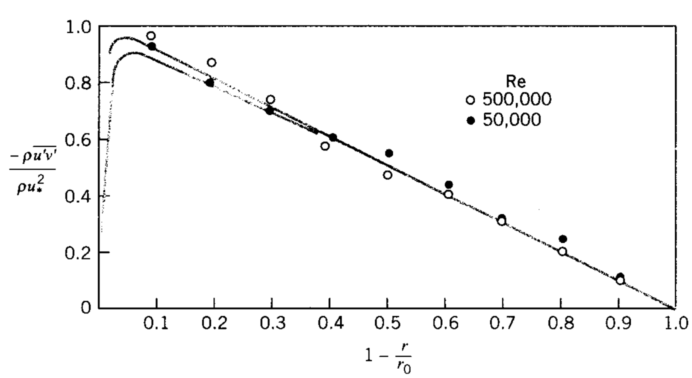
$$u^+ = \frac{1}{\kappa}\ln y^+ + B \qquad 20 < y^+ < 10^5$$

$$\kappa = .41 \qquad B = 5.5$$

3. outer layer (turbulent shear dominates)

$$\frac{U-u}{u^+} = f\left(1-\frac{r}{r_o}\right) \qquad y^+ > 10^5$$

FIGURE 10.4
Apparent shear stress in a pipe. [After Laufer (23)]



Assume log-law is valid across entire pipe

$$u^+ = \sqrt{\frac{\tau_w}{\rho}} = \text{friction velocity}$$

$$\frac{u(r)}{u^*} = \frac{1}{\kappa}\ln\frac{(r_o - r)u^*}{\nu} + B$$

$$\kappa = .41$$

$$B = 5.5$$

$$\overline{V} = \frac{Q}{A} = \frac{\int_0^{r_o} u(r) 2\pi r dr}{\pi r_o^2} = \frac{1}{2}u^*\left\{\frac{2}{\kappa}\ln\frac{r_o u^*}{\nu} + 2B - \frac{3}{\kappa}\right\}$$

drop over bar: $$\frac{V}{u^*} = 2.44\ln\underbrace{\frac{r_o u^*}{\nu}}_{\frac{1}{2}\mathrm{Re}\left(\frac{f}{8}\right)^{1/2}} + 1.34 = \left(\frac{\rho V^2}{\tau_o}\right)^{1/2} = \left(\frac{8}{f}\right)^{1/2}$$

$$\frac{1}{\sqrt{f}} = 1.99\log\left(\mathrm{Re}\, f^{1/2}\right) - 1.02$$

constants adjusted using data $$\Rightarrow \frac{1}{\sqrt{f}} = 2\log\left(\mathrm{Re}\, f^{1/2}\right) - .8 \qquad \mathrm{Re} > 3000$$

Power law $$\Rightarrow f \sim .316\mathrm{Re}^{-1/4} \qquad 4000 < \mathrm{Re} < 10^5$$

$$h_f = -\Delta h = -\left(\frac{\Delta p}{\gamma} + \Delta z\right) = f\frac{L}{D}\frac{V^2}{2g}$$

$$h_f = .316\left(\frac{\mu}{\rho V D}\right)^{1/4}\frac{L}{D}\frac{V^2}{2g}$$

$$h_f \propto V^{1.75}$$

(recall $h_f \propto V$ for laminar flow)

Other useful relationships

Power law fit to velocity profile:

$$\frac{u}{u_{max}} = \left(\frac{y}{r_o}\right)^m \qquad y = r_o - r$$

$$m = m(Re)$$

$$\frac{u_{max}}{u^*} = \frac{1}{\kappa}\ln\frac{r_o u^*}{r} + B$$

$$\frac{V}{u_{max}} = \left(1 + 1.33 f^{1/2}\right)^{-1}$$

TABLE 10.1 EXPONENTS FOR POWER-LAW EQUATION AND RATIO OF MEAN TO MAXIMUM VELOCITY

Re $\rightarrow$	4×10^3	2.3×10^4	1.1×10^5	1.1×10^6	3.2×10^6
$m \rightarrow$	$\frac{1}{6.0}$	$\frac{1}{6.6}$	$\frac{1}{7.0}$	$\frac{1}{8.8}$	$\frac{1}{10.0}$
$\overline{V}/V_{max} \rightarrow$	0.791	0.807	0.817	0.850	0.865

SOURCE: Schlichting (36). Used with permission of the McGraw-Hill Companies.

Viscous Distribution and Resistance – Rough Pipes

For laminar flow, effect of roughness is small; however, for turbulent flow the effect is large. Both laminar sublayer and overlap layer are affected.

Inner layer:

$$u = u(y, k, \rho, \tau_w)$$

$$u^+ = u^+(y/k)$$

not function of μ as was case for smooth pipe (or wall)

Outer layer: unaffected

Overlap layer:

$$u_R^+ = \frac{1}{\kappa}\ln\frac{y}{k} + \text{constant} \qquad \text{rough}$$

$$u_S^+ = \frac{1}{\kappa}\ln y^+ + B \qquad \text{smooth}$$

$$u_S^+ - u_R^+ = \underbrace{\frac{1}{\kappa}\ln k^+ + \text{constant}}_{\Delta B(k^+)} \qquad k^+ = \frac{ku^*}{\nu}$$

i.e., rough-wall velocity profile shifts downward by $\Delta B(k^+)$, which increases with k^+.

three regions of flow depending on k^+

1. $k^+ < 5$ hydraulically smooth (no effect of roughness)
2. $5 < k^+ < 70$ transitional roughness (Re dependence)
3. $k^+ > 70$ fully rough (independent Re)

For 3, $\Delta B = \frac{1}{\kappa}\ln k^+ - 3.5$ from data

$$u^+ = \frac{1}{\kappa}\ln\frac{y}{k} + 8.5 \neq f(\mathrm{Re})$$

$$\frac{V}{u^*} = 2.44\ln\frac{D}{k} + 3.2$$

$$\frac{1}{f^{1/2}} = -2\log\frac{k/D}{3.7}$$

fully rough flow

<u>Composite Log-Law</u>

$$u^+ = \overbrace{\frac{1}{\kappa}\ln y^+}^{\text{Smooth wall log law}} + \underbrace{B - \Delta B(k^+)}_{B^*}$$

$$B^* = 5 - \frac{1}{\kappa}\ln(1 + .3k^+)$$ from data

$$\frac{1}{f^{1/2}} = -2\log\left[\frac{k/D}{3.7} + \frac{2.51}{\mathrm{Re}\,f^{1/2}}\right]$$ Moody Diagram

$$= 1.14 - 2\log\left(\frac{k_s}{D} + \frac{9.35}{\mathrm{Re}\,f^{1/2}}\right)$$

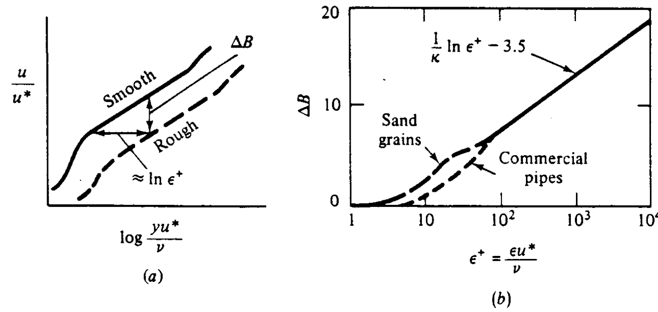


Fig. 6.12 Effect of wall roughness on turbulent pipe-flow velocity profiles: (*a*) logarithm-law downshift; (*b*) correlation with roughness

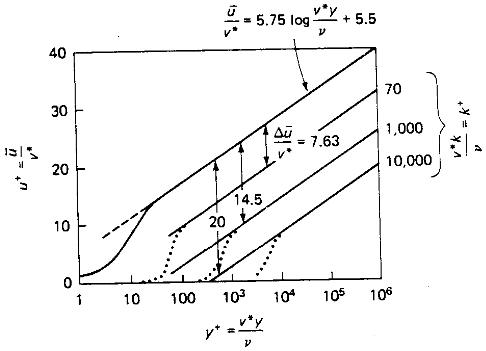


FIGURE 6-11
Experimental rough-pipe velocity profiles by Scholz (1955), showing the ·nward shift ΔB of the logarithmic overlap layer.

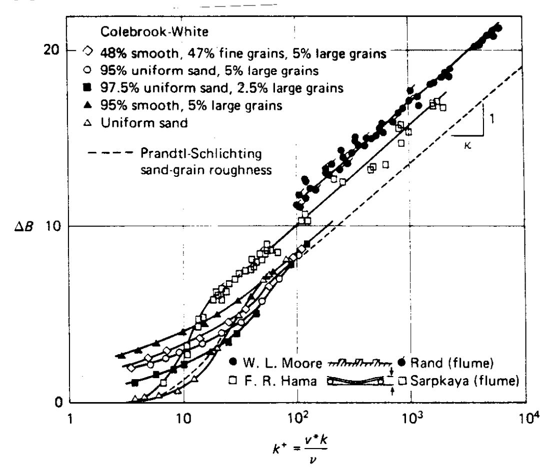


FIGURE 6-12
Composite plot of the profile-shift parameter $\Delta B(k^+)$ for various roughness geometries, as compiled by Clauser (1956).

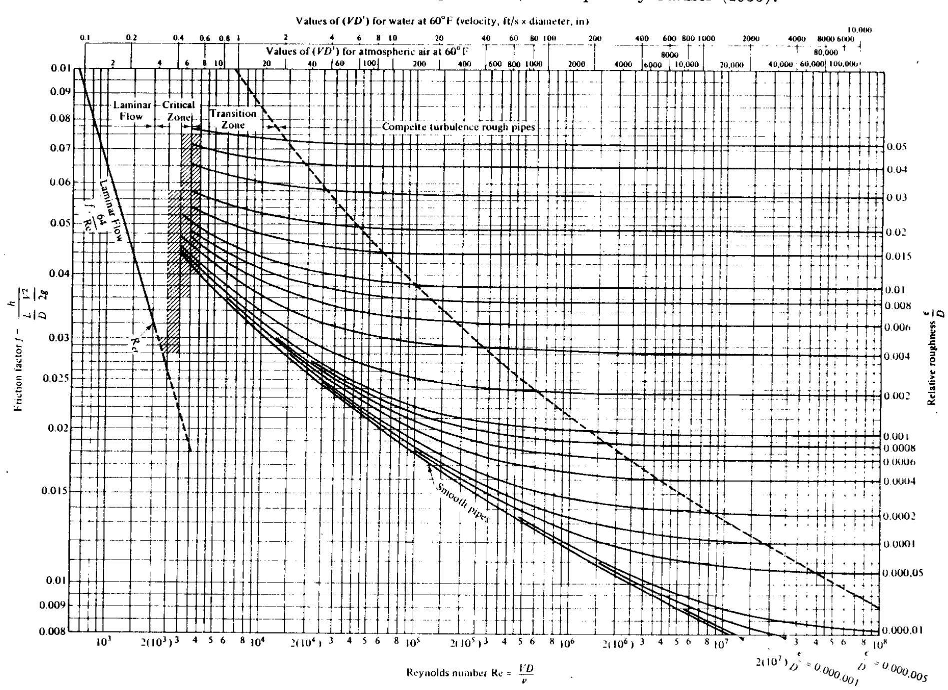


Fig. 6.13 The Moody chart for pipe friction with smooth and rough walls. (*From Ref. 8, by permission of the ASME.*)

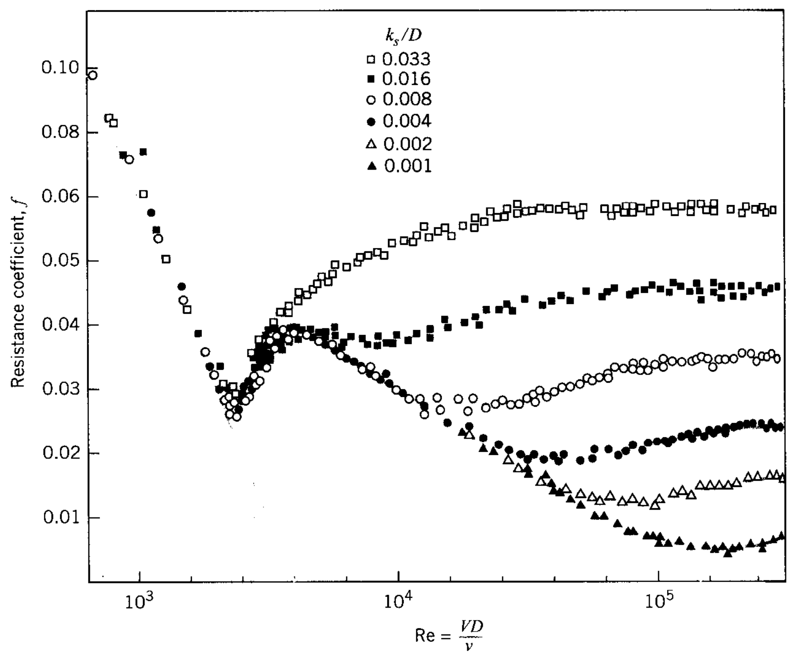
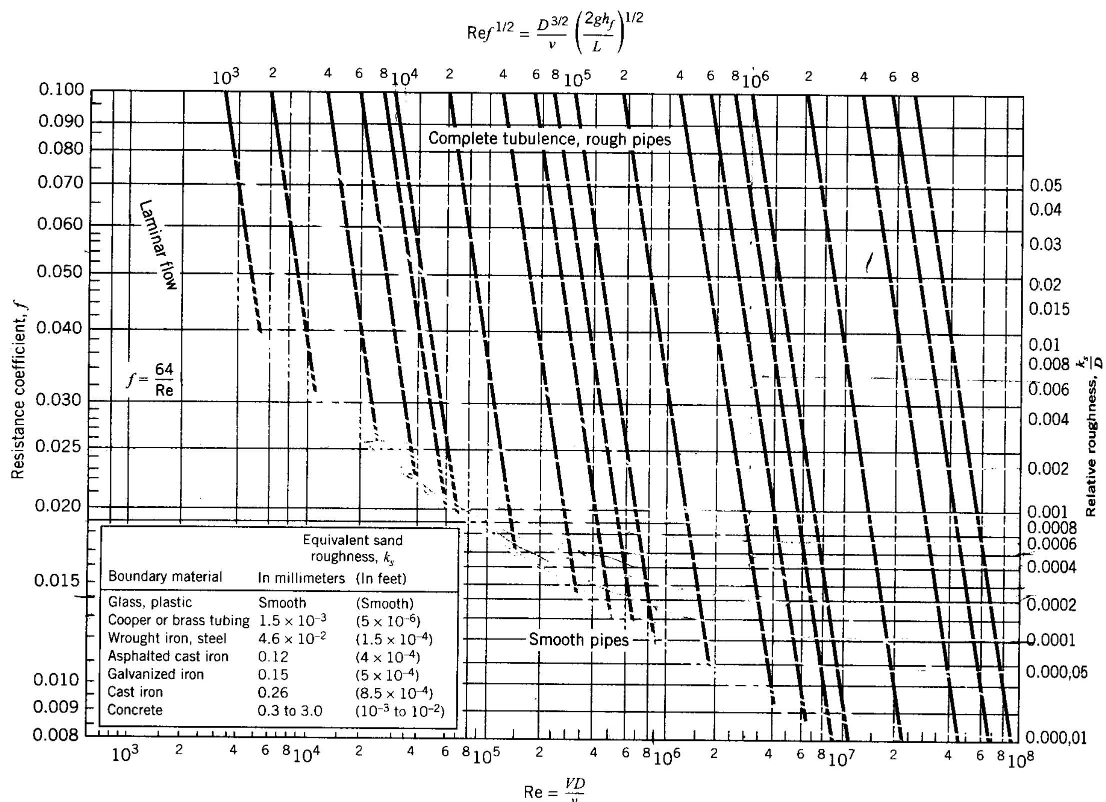


FIGURE 10.7

Resistance coefficient f *versus Re for sand-roughened pipe. [After Nikuradse (30)]*



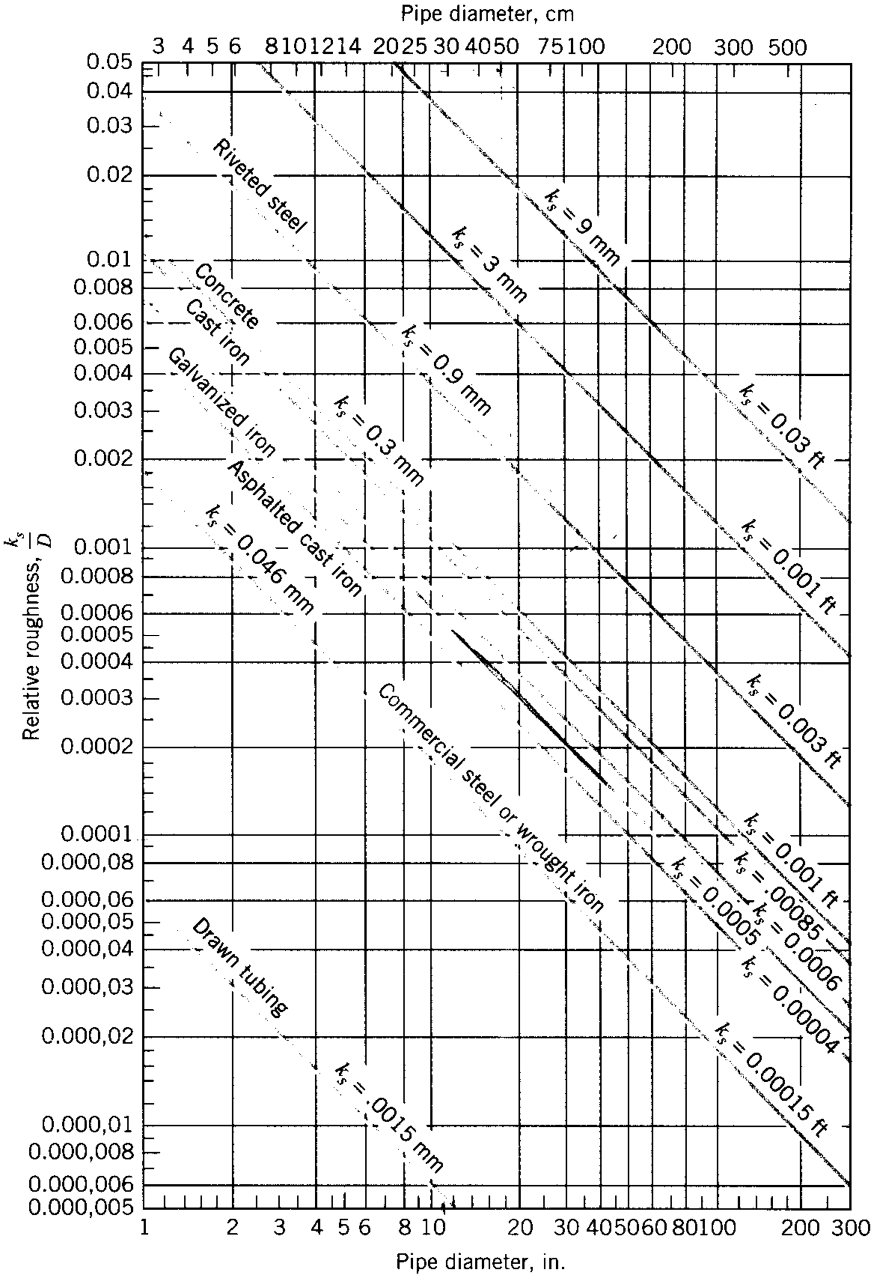
Boundary material	Equivalent sand roughness, k_s In millimeters	(In feet)
Glass, plastic	Smooth	(Smooth)
Cooper or brass tubing	1.5×10^{-3}	(5×10^{-6})
Wrought iron, steel	4.6×10^{-2}	(1.5×10^{-4})
Asphalted cast iron	0.12	(4×10^{-4})
Galvanized iron	0.15	(5×10^{-4})
Cast iron	0.26	(8.5×10^{-4})
Concrete	0.3 to 3.0	$(10^{-3}$ to $10^{-2})$

FIGURE 10.8

Resistance coefficient f *versus Re. Reprinted with minor variations. [After Moody (29). Reprinted with permission from the A.S.M.E.]*

FIGURE 10.9

Relative roughness for various kinds of pipe. [After Moody (29). Reprinted with permission from the A.S.M.E.]



There are basically three types of problems involved with uniform flow in a single pipe:

1. Determine the head loss, given the kind and size of pipe along with the flow rate, Q = A*V
2. Determine the flow rate, given the head, kind, and size of pipe
3. Determine the pipe diameter, given the type of pipe, head, and flow rate

1. Determine the head loss
   The first problem of head loss is solved readily by obtaining f from the Moody diagram, using values of Re and k_s/D computed from the given data. The head loss h_f is then computed from the Darcy-Weisbach equation.

$$f = f(Re_d, k_s/D)$$

$$h_f = f\frac{L}{D}\frac{V^2}{2g} = -\Delta h \qquad \Delta h = \left(\frac{p_2}{\gamma} + z_2\right) - \left(\frac{p_1}{\gamma} + z_1\right)$$

$$= -\Delta\left(\frac{p}{\gamma} + z\right)$$

$$Re_d = Re_d(V, D)$$

2. Determine the flow rate
The second problem of flow rate is solved by trial, using a successive approximation procedure. This is because both Re and f(Re) depend on the unknown velocity, V. The solution is as follows:

1) solve for V using an assumed value for f and the Darcy-Weisbach equation

$$V = \underbrace{\left[\frac{2gh_f}{L/D}\right]^{1/2}}_{\text{known from given data}} \cdot f^{-1/2}$$

note sign

2) using V compute Re
3) obtain a new value for f = f(Re, k_s/D) and reapeat as above until convergence

Or can use $Re = f^{1/2} = \frac{D^{3/2}}{\nu}\left(\frac{2gh_f}{L}\right)^{1/2}$ scale on Moody Diagram

1) compute $Re f^{1/2}$ and k_s/D
2) read f
3) solve V from $h_f = f \frac{L}{D}\frac{V^2}{2g}$
4) Q = VA

3. Determine the size of the pipe
   The third problem of pipe size is solved by trial, using a successive approximation procedure. This is because h_f, f, and Q all depend on the unknown diameter D. The solution procedure is as follows:

   1) solve for D using an assumed value for f and the Darcy-Weisbach equation along with the definition of Q

$$D = \underbrace{\left[\frac{8LQ^2}{\pi^2 g h_f}\right]^{1/5}}_{\text{known from given data}} \cdot f^{1/5}$$

   2) using D compute Re and k_s/D

   3) obtain a new value of $f = f(Re, k_s/D)$ and reapeat as above until convergence

8.5 Flow at Pipe Inlets and Losses From Fittings

For real pipe systems in addition to friction head loss these are additional so called minor losses due to

1. entrance and exit effects
2. expansions and contractions
3. bends, elbows, tees, and other fittings
4. valves (open or partially closed)

can be large effect

For such complex geometries we must rely on experimental data to obtain a loss coefficient

$$K = \frac{h_m}{\frac{V^2}{2g}}$$

head loss due to minor losses

In general,

$$K = K(\text{geometry}, \underbrace{Re, \varepsilon/D}_{\text{dependence usually not known}})$$

Loss coefficient data is supplied by manufacturers and also listed in handbooks. The data are for turbulent flow conditions but seldom given in terms of Re.

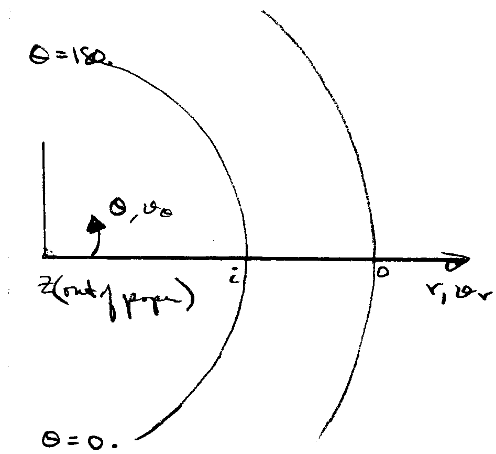
Modified Energy Equation to Include Minor Losses:

$$\frac{p_1}{\gamma}+z_1+\frac{1}{2g}\alpha_1 V_1^2+h_p=\frac{p_2}{\gamma}+z_2+\frac{1}{2g}\alpha_2 V_2^2+h_t+h_f+\sum h_m$$

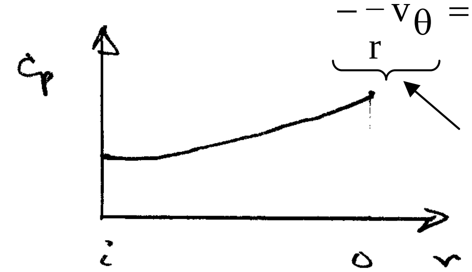
$$h_m = K\frac{V^2}{2g}$$

Note: Σh_m does not include pipe friction and e.g. in elbows and tees, this must be added to h_f.

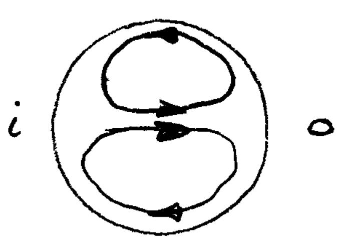
1. Flow in a bend:



$$\underbrace{-\frac{1}{r}v_\theta^2}_{\text{centrifugal acceleration}} = -\frac{1}{\rho}\frac{\partial p}{\partial r}$$

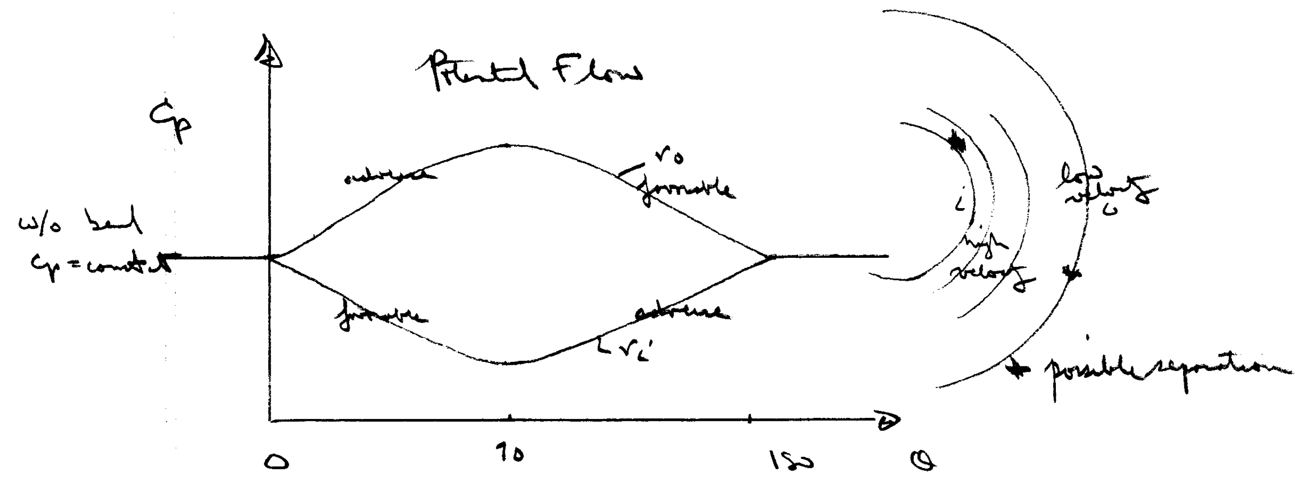


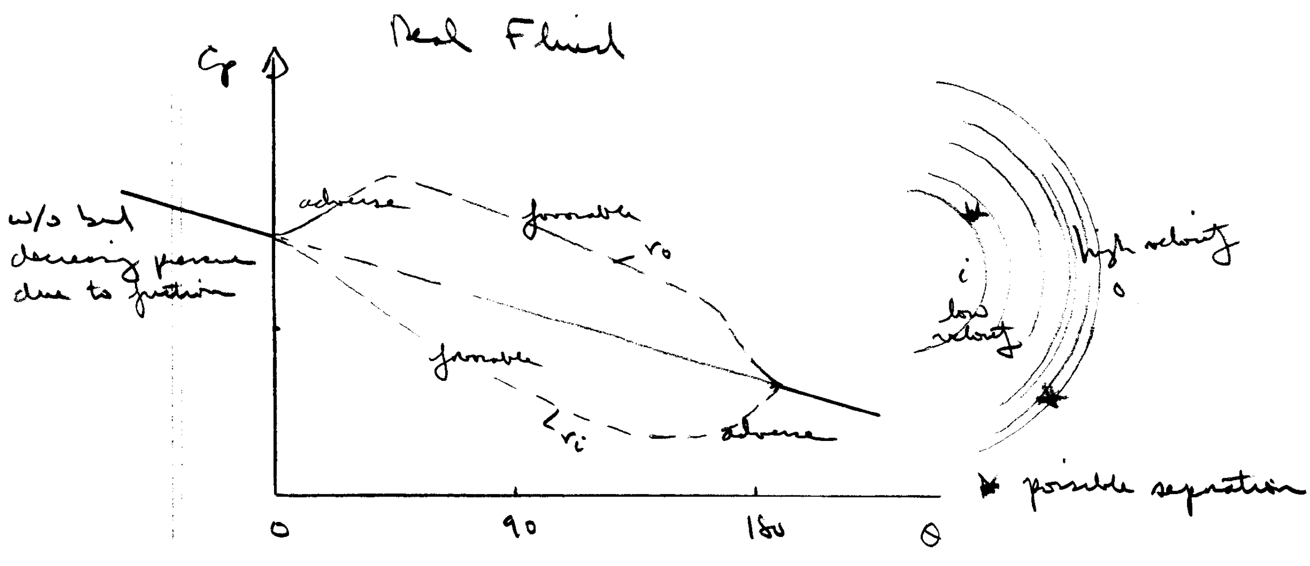
i.e. $\frac{\partial p}{\partial r} > 0$ which is an adverse pressure gradient in r direction. The slower moving fluid near wall responds first and a swirling flow pattern results.



This swirling flow represents an energy loss which must be added to the h_L.

Also, flow separation can result due to adverse longitudinal pressure gradients which will result in additional losses.





This shows potential flow is not a good approximate in internal flows (except possibly near entrance)

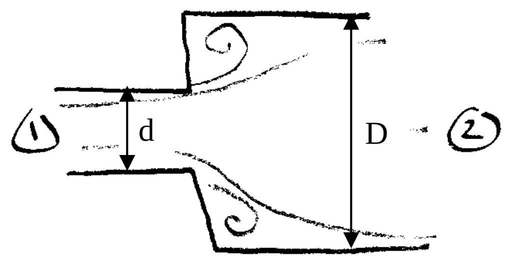
2. Valves: enormous losses

3. Entrances: depends on rounding of entrance

4. Exit (to a large reservoir): K = 1
i.e., all velocity head is lost

5. Contractions and Expansions
$\underbrace{\text{sudden}}$ or gradual

theory for expansion:



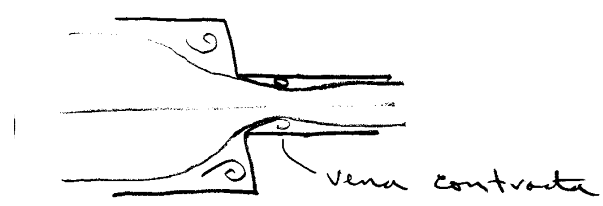
$$h_L = \frac{(V_1 - V_2)^2}{2g}$$

from continuity, momentum, and energy
(assuming $p = p_1$ in separation pockets)

$$\Rightarrow \quad K_{SE} = \left(1 - \frac{d^2}{D^2}\right)^2 = \frac{h_m}{V_1^2/2g}$$

no theory for contraction:

$$K_{SC} = .42\left(1 - \frac{d^2}{D^2}\right)$$



from experiment

If the contraction or expansion is gradual the losses are quite different. A gradual expansion is called a diffuser. Diffusers are designed with the intent of raising the static pressure.

$$C_p = \frac{p_2 - p_1}{\frac{1}{2}\rho V_1^2}$$

$$C_{p_{ideal}} = 1 - \left(\frac{A_1}{A_2}\right)^2$$ Bernoulli and continuity equation

$$K = \frac{h_m}{V^2/2g} = C_{p_{ideal}} - C_p$$ Energy equation

Actually very complex flow and

$C_p = C_p$ (geometry, $\underbrace{\text{inlet flow conditions}}$)

i.e., fully developed (long pipe) reduces C_p
thin boundary layer (short pipe) high C_p
(more uniform inlet profile)

FIGURE 10.10
Flow characteristics at a pipe inlet (not to scale).

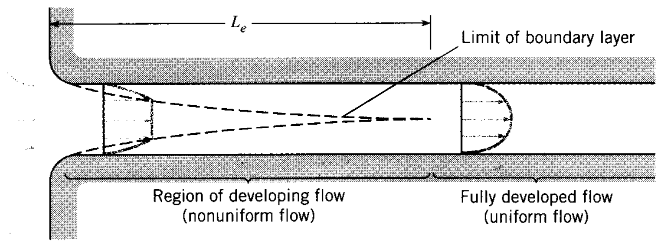
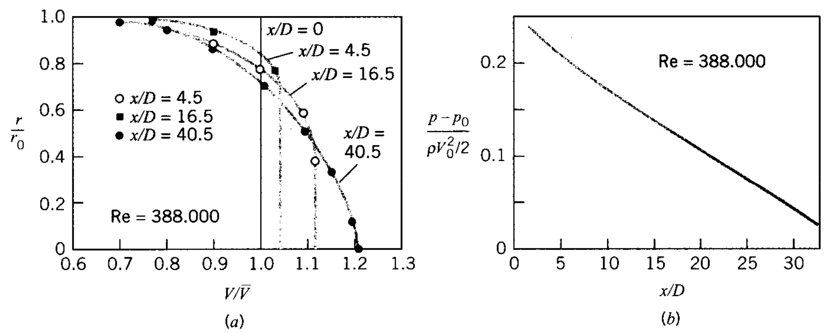
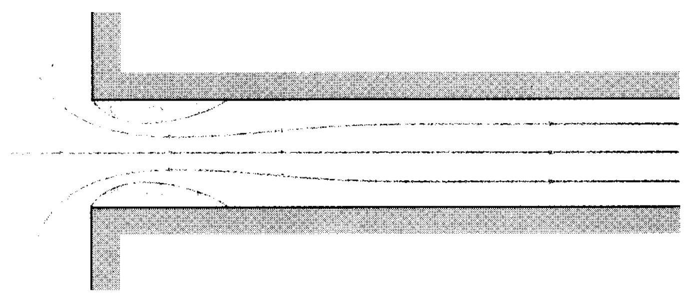


FIGURE 10.11
Distribution of velocity and pressure in the inlet region of a pipe [Barbin and Jones (3)].
(a) Velocity distribution.
(b) Pressure distribution.



Turbulent flow

FIGURE 10.12
Flow at a sharp-edged inlet.



K = .5

FIGURE 10.13
Flow pattern in an elbow.

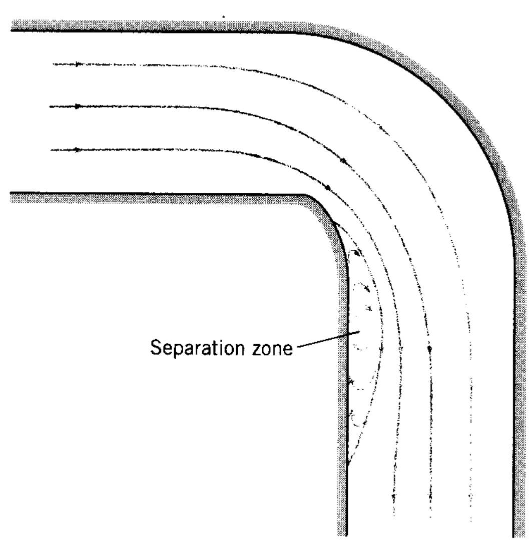


TABLE 10.2 LOSS COEFFICIENTS FOR VARIOUS TRANSITIONS AND FITTINGS

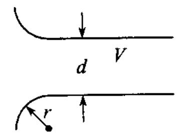
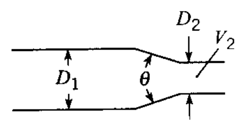
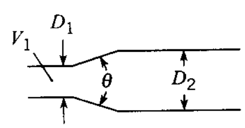
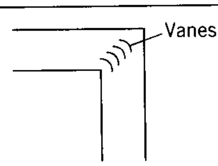
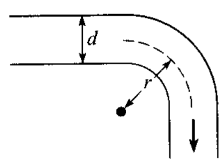
Description	Sketch	Additional Data	K		Source
		r/d	K_e		(2)*
Pipe entrance		0.0	0.50		
		0.1	0.12		
$h_L = K_e V^2/2g$		>0.2	0.03		
			K_C	K_C	
Contraction		D_2/D_1	$\theta = 60°$	$\theta = 180°$	(2)
		0.0	0.08	0.50	
		0.20	0.08	0.49	
		0.40	0.07	0.42	
		0.60	0.06	0.27	
		0.80	0.06	0.20	
$h_L = K_C V_2^2/2g$		0.90	0.06	0.10	
			K_E	K_E	
Expansion		D_1/D_2	$\theta = 20°$	$\theta = 180°$	(2)
		0.0		1.00	
		0.20	0.30	0.87	
		0.40	0.25	0.70	
		0.60	0.15	0.41	
$h_L = K_E V_1^2/2g$		0.80	0.10	0.15	
90° miter bend		Without vanes	$K_b = 1.1$		(37)
		With vanes	$K_b = 0.2$		(37)
90° smooth bend		r/d			(5) and (19)
		1	$K_b = 0.35$		
		2	0.19		
		4	0.16		
		6	0.21		
		8	0.28		
		10	0.32		
Threaded pipe fittings		Globe valve—wide open	$K_v = 10.0$		(37)
		Angle valve—wide open	$K_v = 5.0$		
		Gate valve—wide open	$K_v = 0.2$		
		Gate valve—half open	$K_v = 5.6$		
		Return bend	$K_b = 2.2$		
		Tee			
		straight-through flow	$K_t = 0.4$		
		side-outlet flow	$K_t = 1.8$		
		90° elbow	$K_b = 0.9$		
		45° elbow	$K_b = 0.4$		

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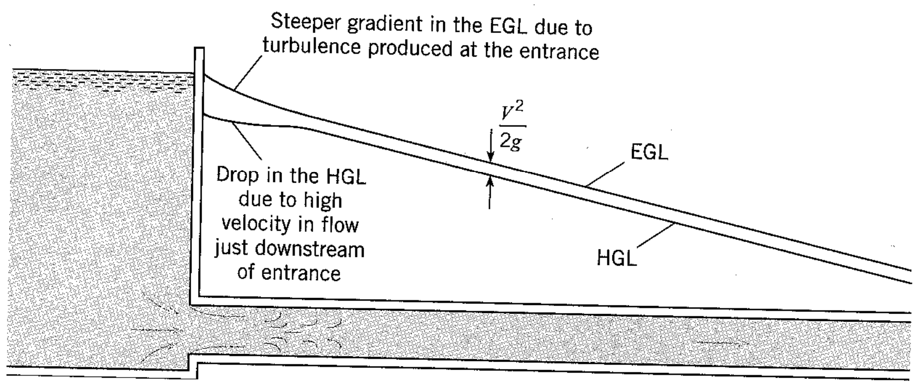
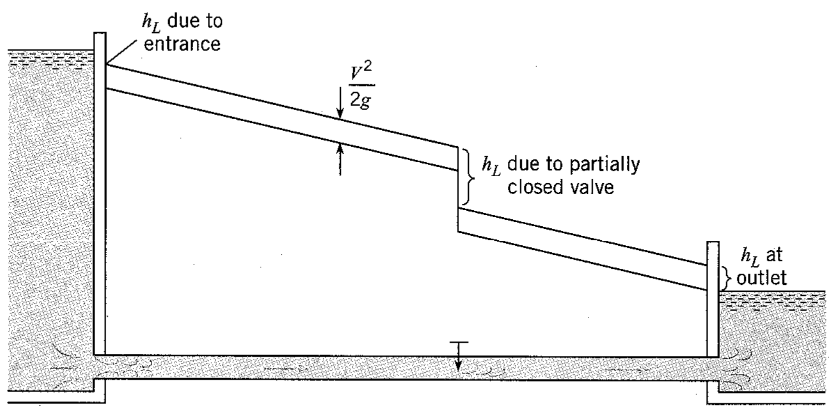
FIGURE 10.14
EGL and HGL at a sharp-edged pipe entrance.





IGURE 10.15
ead losses in a pipe.