4.5 Separation, Vortices, Turbulence, and Flow Classification

We will take this opportunity and expand on the material provided in the text to give a general discussion of fluid flow classifications and terminology.

1. One-, Two-, and Three-dimensional Flow

   1D: $\underline{V} = u(y)\hat{i}$

   2D: $\underline{V} = u(x, y)\hat{i} + v(x, y)\hat{j}$

   3D: $\underline{V} = \underline{V}(\underline{x}) = u(x, y, z)\hat{i} + v(x, y, z)\hat{j} + w(x, y, z)\hat{k}$

2. Steady vs. Unsteady Flow

   $\underline{V} = \underline{V}(\underline{x},t)$ unsteady flow

   $\underline{V} = \underline{V}(\underline{x})$ steady flow

3. Incompressible and Compressible Flow

   $\frac{D\rho}{Dt} = 0 \Rightarrow$ incompressible flow

   $Ma = \frac{V}{c}$

   V: representative velocity

   c: speed of sound in fluid

$Ma < .3$	incompressible
$Ma > .3$	compressible
$Ma = 1$	sonic (commercial aircraft Ma~.8)
$Ma > 1$	supersonic

Ma is the most important nondimensional parameter for compressible flow (Chapter 8 Dimensional Analysis)

4. Viscous and Inviscid Flows

Inviscid flow: ($\mu = 0$)	neglect μ, which simplifies analysis but must decide when this is a good approximation (D' Alembert paradox body in steady motion $C_D = 0$!)
Viscous flow: ($\mu \neq 0$)	retain μ, i.e., "Real-Flow Theory" more complex analysis, but often no choice

5. Rotational vs. Irrotational Flow

$\underline{\Omega} = \nabla \times \underline{V} \neq 0$ rotational flow

$\underline{\Omega} = 0$ irrotational flow

Generation of vorticity usually is the result of viscosity $\therefore$ viscous flows are always rotational, whereas inviscid flows are usually irrotational. Inviscid, irrotational, incompressible flow is referred to as ideal-flow theory.

6. Laminar vs. Turbulent Viscous Flows
   Laminar flow = smooth orderly motion composed of thin sheets (i.e., laminas) gliding smoothly over each other

   Turbulent flow = disorderly high frequency fluctuations superimposed on main motion. Fluctuations are visible as eddies which continuously mix, i.e., combine and disintegrate (average size is referred to as the scale of turbulence).

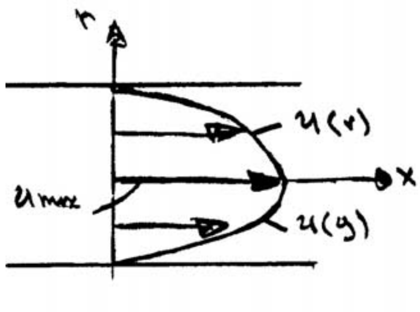
Re - decomposition

$$u = \overline{u} + u'(t)$$

mean motion ($\overline{u}$); turbulent fluctuation ($u'(t)$)

usually $u' \sim (.01\text{-}.1)\overline{u}$, but influence is as if μ increased by 100-10,000 or more.

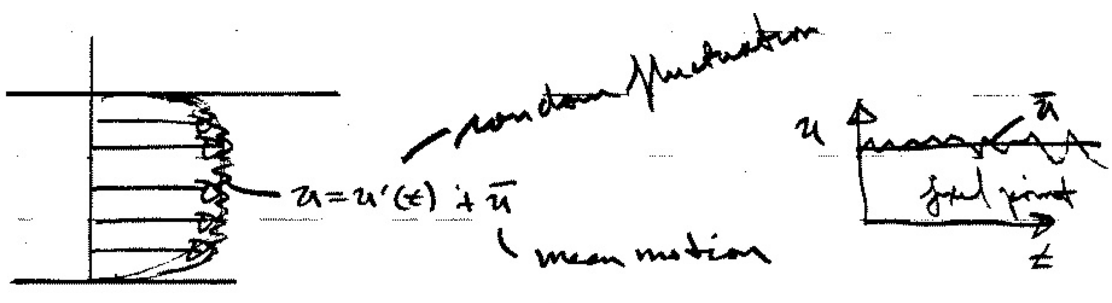
Example: Pipe Flow (Chapter 10 = Flow in Conduits)
Laminar flow:



$$u(r) = \frac{R^2 - r^2}{4\mu}\left(-\frac{dp}{dx}\right)$$

u(y),velocity profile in a paraboloid

Turbulent flow: fuller profile due to turbulent mixing extremely complex fluid motion that defies closed form analysis.



Turbulent flow is the most important area of motion fluid dynamics research.

The most important nondimensional number for describing fluid motion is the Reynolds number (Chapter 8)

$$Re = \frac{VD\rho}{\mu} = \frac{VD}{\nu}$$

V = characteristic velocity
D = characteristic length

For pipe flow

$V = \overline{V}$ = average velocity

D = pipe diameter

Re < 2000 laminar flow

Re > 2000 turbulent flow

Also depends on roughness, free-stream turbulence, etc.

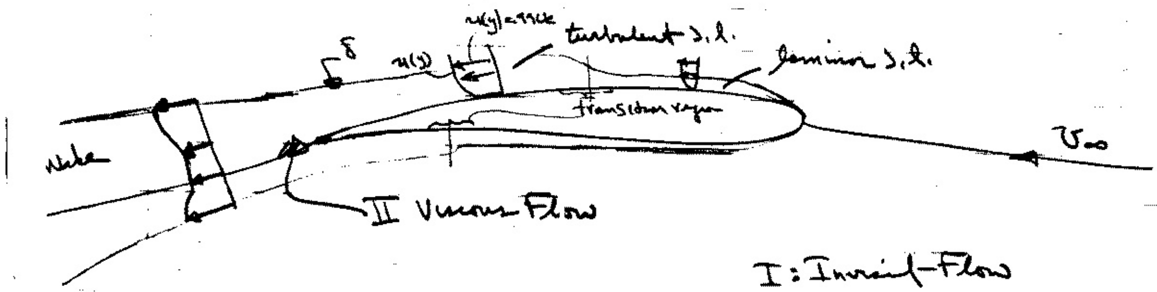
7. Internal vs. External Flows

Internal flows = completely wall bounded;
Usually requires viscous analysis, except near entrance (Chapter 10)

External flows = unbounded; i.e., at some distance from body or wall flow is uniform (Chapter 9, Surface Resistance)

External Flow exhibits flow-field regions such that both inviscid and viscous analysis can be used depending on the body shape and Re.

Flow Field Regions (high Re flows)



$$\mathrm{Re} = \frac{Vc}{\nu} = \frac{\text{inertia force}}{\text{viscous force}}$$

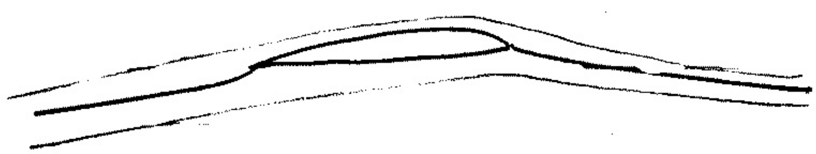
Important features:

1) low Re viscous effects important throughout entire fluid domain: creeping motion
2) high Re flow about streamlined body viscous effects confined to narrow region: boundary layer and wake
3) high Re flow about bluff bodies: in regions of adverse pressure gradient flow is susceptible to separation and viscous-inviscid interaction is important

8. Separated vs. Unseparated Flow

Streamlined body

Flow remains attached w/o separation

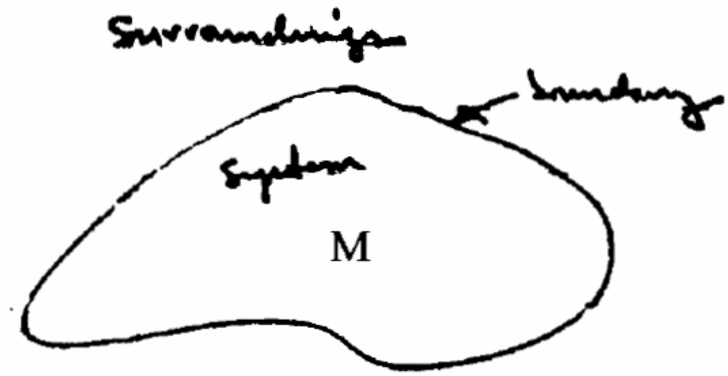


Bluff body

Flow separates and creates the region of reverse flow, i.e. separation

4.6 Basic Control-Volume Approach and RTT

Laws of mechanics are written for a system, i.e., a fixed amount of matter.



1. Conservation of mass: $\frac{dM}{dt} = 0$
2. Conservation of momentum: $\underline{F} = M\underline{a} = \frac{d(M\underline{V})}{dt}$
3. Conservation of energy: $\frac{dE}{dt} = \dot{Q} - \dot{W}$

   ΔE=heat added – work done

Also

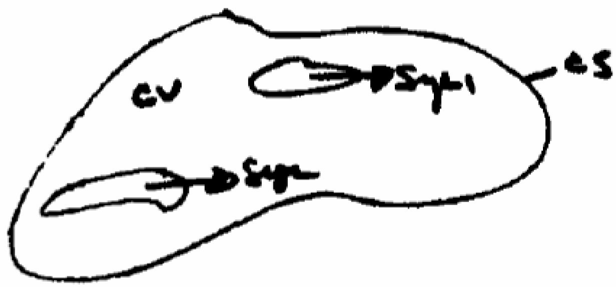
Conservation of angular momentum: $\frac{d\underline{H}_G}{dt} = \underline{M}_G$

Second Law of Thermodynamics: $\frac{dS}{dt} = \frac{\delta\dot{Q}}{T} + \dot{\sigma}$

$\dot{\sigma}$, entropy production due to system irreversibilities

$\dot{\sigma} \le 0$

In fluid mechanics we are usually interested in a region of space, i.e, control volume and not particular systems. Therefore, we need to transform GDE's from a system to a control volume, which is accomplished through the use of



RTT (actually derived in thermodynamics for CV forms of continuity and 1st and 2nd laws, but not in general form or referred to as RTT).

Note GDE's are of form:

$$\frac{d}{dt}\underbrace{(M, M\underline{V}, E)}_{\text{system extensive properties } B_{sys} \text{ depend on mass}} = RHS$$

i.e., involve $\frac{dB_{sys}}{dt}$ which needs to be related to changes in CV. Recall, definition of corresponding system intensive properties

$\beta = (1, \underline{V}, e)$ independent of mass

where

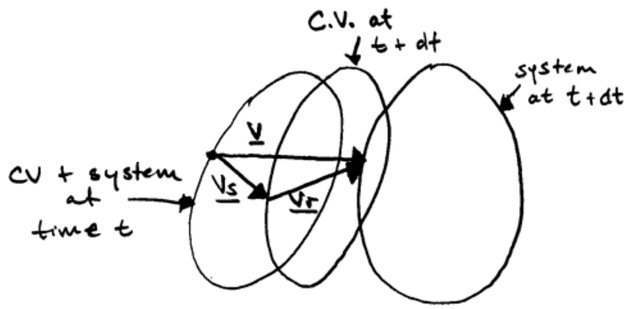
$$B = \int \beta dm = \int \beta \rho d\forall$$

i.e., $\beta = \frac{dB}{dm}$

Reynolds Transport Theorem (RTT)

Need relationship between $\frac{d}{dt}\left(B_{sys}\right)$ and changes in

$B_{CV} = \int\limits_{CV} \beta\, dm = \int\limits_{CV} \beta\rho\, d\forall$.



Moving deforming CV:
$\underline{V}_r = \underline{V} - \underline{V}_s$

$\underline{V}$ = fluid velocity
$\underline{V}_s$ = CS defining CV velocity
$\underline{V}_r$ = relative velocity

} all in same coordinate system

$$\frac{dB_{sys}}{dt} = \lim_{\Delta t \to 0} \frac{(B_{cv} + \Delta B)_{t+\Delta t} - (B_{cv} + \Delta B)_t}{\Delta t}$$

$$= \underbrace{\lim_{\Delta t \to 0} \frac{B_{cv_{t+\Delta t}} - B_{cv_t}}{\Delta t}}_{①} + \underbrace{\lim_{\Delta t \to 0} \frac{\Delta B_{t+\Delta t} - \Delta B_t}{\Delta t}}_{②}$$

1 = time rate of change of B in CV $= \frac{dB_{cv}}{dt} = \frac{d}{dt} \int\limits_{CV} \beta\rho\, d\forall$

2 = net outflux of B from CV across CS = $\int\limits_{CS} \beta\rho\, \underline{v}_R \cdot \underline{n}\, DA$

$$\frac{dB_{SYS}}{dt} = \frac{d}{dt} \int\limits_{CV} \beta\rho\, d\forall + \int\limits_{CS} \beta\rho \underline{v}_R \cdot \underline{n}\, dA$$

General form RTT for moving deforming control volume

Special Cases:

1) Non-deforming CV moving at constant velocity

$$\frac{dB_{SYS}}{dt} = \int_{CV} \frac{\partial}{\partial t}(\beta\rho)d\forall + \int_{CS} \beta\rho \underline{v}_R \cdot \underline{n}\, dA$$

2) Fixed CV

$$\frac{dB_{SYS}}{dt} = \int_{CV} \frac{\partial}{\partial t}(\beta\rho)d\forall + \int_{CS} \beta\rho \underline{v} \cdot \underline{n}\, dA$$

Greens Theorem: $\int_{CV} \nabla \cdot \underline{b}\, d\forall = \int_{CS} \underline{b} \cdot \underline{n}\, dA$

$$\frac{dB_{SYS}}{dt} = \int_{CV} \left[\frac{\partial}{\partial t}(\beta\rho) + \nabla \cdot (\beta\rho\underline{v})\right] d\forall$$

Since CV fixed and arbitrary $\lim_{d\forall \to 0}$ gives differential eq.

3) Steady Flow: $\frac{\partial}{\partial t} = 0$

4) Uniform flow across discrete CS (steady or unsteady)

$$\int_{CS} \beta\rho\, \underline{v} \cdot \underline{n}\, dA = \sum_{CS} \beta\rho\, \underline{v} \cdot \underline{n}\, dA \quad \textit{(- inlet, + outlet)}$$

Continuity Equation:

B = M = mass of system
β = L

$\frac{dM}{dt} = 0$ by definition, system = fixed amount of mass

Integral Form:

$$\frac{dM}{dt} = 0 = \frac{d}{dt}\int_{CV} \rho \, d\forall + \int_{CS} \rho \underline{v}_R \cdot \underline{n} \, dA$$

$$-\frac{d}{dt}\int_{CV} \rho \, d\forall = \int_{CS} \rho \underline{v}_R \cdot \underline{n} \, dA$$

Rate of decrease of mass in CV = net rate of mass outflow across CS

Note simplifications for non-deforming CV, fixed CV, steady flow, and uniform flow across discrete CS

Incompressible Fluid: ρ = constant

$$-\frac{d}{dt}\int_{CV} d\forall = \int_{CS} \underline{v}_R \cdot \underline{n} \, dA$$

"conservation of volume"

Differential Form:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \underline{v}) = 0$$

$$M = \rho \, d\forall$$

$$dM = \rho \, d\forall + \forall \, d\rho = 0$$

$$-\frac{d\forall}{\forall} = \frac{d\rho}{\rho}$$

$$\frac{\partial \rho}{\partial t} + \rho \nabla \cdot \underline{v} + \underline{v} \cdot \nabla \rho = 0$$

$$\frac{1}{\rho}\frac{D\rho}{Dt} = -\frac{1}{\forall}\frac{D\forall}{Dt}$$

$$\frac{D\rho}{Dt} + \rho \nabla \cdot \underline{v} = 0$$

$$\underbrace{\frac{1}{\rho}\frac{D\rho}{Dt}}_{\text{rate of change } \rho \text{ per unit } \rho} + \underbrace{\nabla \cdot \underline{v}}_{\underbrace{\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}}_{\text{rate of change } \forall \text{ per unit } \forall}} = 0$$

Called the continuity equation since the implication is that ρ and $\underline{v}$ are continuous functions of $\underline{x}$.

Incompressible Fluid: ρ = constant

$$\nabla \cdot \underline{v} = 0$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$