Chapter 2: Properties of Fluids

Fluids are defined by their properties such as viscosity μ and density ρ, which we have already discussed with reference to definition of shear stress $\tau = \mu\dot{\theta}$ and the continuum hypothesis.

Properties can be both dimensional (i.e., expressed in either SI or BG units) or non-dimensional.

See: Figure 2-17 and Appendix Tables A-3, A-3E, A-7, A-7E, A-9, and A-9E

2.1 Basic Units

System International and British Gravitational Systems

Primary Units	SI	BG
Mass M	kg	Slug=32.2lbm
Length L	m	ft
Time t	s	s
Temperature T	°C (°K)	°F (°R)

Temperature Conversion:

°K = °C + 273

°R = °F + 460

°K and °R are absolute scales, i.e., 0 at absolute zero. Freezing point of water is at 0°C and 32°F.

Secondary (derived) units	Dimension	SI	BG
velocity V	L/t	m/s	ft/s
acceleration a	L/t^2	m/s^2	ft/s^2
force F	ML/t^2	N ($kg\cdot m/s^2$)	lbf
pressure p	F/L^2	Pa (N/m^2)	lbf/ft^2
density ρ	M/L^3	kg/m^3	$slug/ft^3$
internal energy u	FL/M	J/kg (N·m/kg)	BTU/lbm

Weight and Mass

$\mathbf{F} = m\mathbf{a}$ Newton's second law (valid for both solids and fluids)

Weight = force on object due to gravity

W = mg $g = 9.81 \text{ m/s}^2$
$= 32.2 \text{ ft/s}^2$

SI: $W\,(N) = M\,(kg) \cdot 9.81 \text{ m/s}^2$

BG: $W\,(lbf) = \dfrac{M(lbm)}{g_c} \cdot 32.2 \text{ ft/s}^2 = M(slug) \cdot 32.2 \text{ ft/s}^2$

$$g_c = 32.2 \frac{lbm \cdot ft}{s^2 \cdot lbf} = 32.2 \frac{lbm}{slug}$$, i.e., 1 slug = 32.2 lbm

$1N = 1kg \cdot 1m/s^2$

$1lbf = 1\ slug \cdot 1ft/s^2$

2.2 System; Extensive and Intensive Properties

System = fixed amount of matter
= mass M

Therefore, by definition

$$\frac{d(M)}{dt} = 0$$

Properties are further distinguished as being either extensive or intensive.

Extensive properties: depend on total mass of system, e.g., M and W (upper case letters)

Intensive properties: independent of amount of mass of system, e.g., p (force/area, lower case letters) and ρ (mass/volume)

2.3 Properties Involving the Mass or Weight of the Fluid

Specific Weight, γ = gravitational force, i.e., weight per unit volume $\forall$

$$= W/\forall$$
$$= mg/\forall$$
$$= \rho g \qquad N/m^3$$

(Note that specific properties are extensive properties per unit mass or volume)

Mass Density ρ = mass per unit volume
= M/V kg/m^3

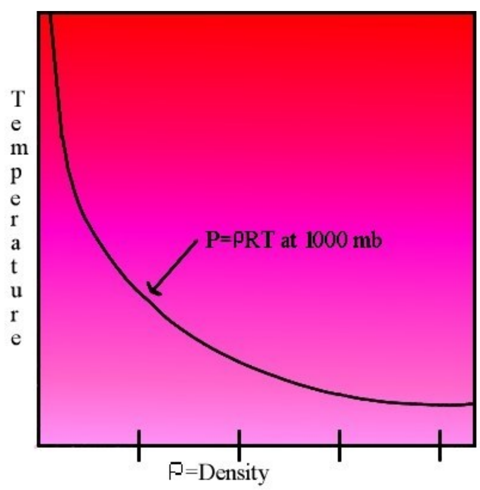
Specific Gravity S = ratio of γ_{fluid} to γ_{water} at standard T = 4°C
= $\gamma/\gamma_{water,\ 4°C}$ dimensionless

$\gamma_{water,\ 4°C}$ = 9810 N/m^3 for T = 4°C and atmospheric pressure

Variation in Density

gases: ρ = ρ (gas, T, p) equation of state (p-v-T)
= p/RT ideal gas

R = R (gas)
R (air) = 287.05 N·m/kg·°K



Air

T	ρ
240	1.45
250	1.39
260	1.34
270	1.29
280	1.24
290	1.20
300	1.16
310	1.12

ρ is in kg m^{-3}
T is in °K

liquids: ρ ~ constant

Liquid and temperature	Density (kg/m^3)	Density ($slugs/ft^3$)
Water 20°C (68°F)	998	1.94
Ethyl alcohol 20°C (68°F)	799	1.55
Glycerine 20°C (68°F)	1,260	2.45
Kerosene 20°C (68°F)	814	1.58
Mercury 20°C (68°F)	13,350	26.3
Sea water 10°C at 3.3% salinity	1,026	1.99
SAE 10W 38°C(100°F)	870	1.69
SAE 10W-30 38°C(100°F)	880	1.71
SAE 30 38°C(100°F)	880	1.71

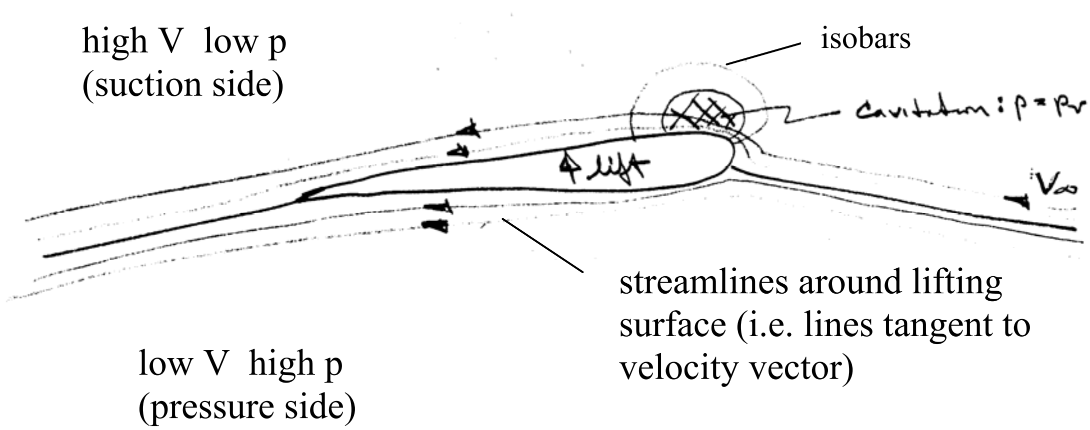
For greater accuracy can also use p-v-T diagram

$$\rho = \rho \text{ (liquid, T, p)}$$

$T\uparrow \qquad \rho\downarrow$
$p\uparrow \qquad \rho\uparrow$

2.4 <u>Vapor Pressure and Cavitation</u>

When the pressure of a liquid falls below the vapor pressure it evaporates, i.e., changes to a gas. If the pressure drop is due to temperature effects alone, the process is called boiling. If the pressure drop is due to fluid velocity, the process is called cavitation. Cavitation is common in regions of high velocity, i.e., low p such as on turbine blades and marine propellers.



$$\text{Cavitation number} = \frac{p - p_v}{\frac{1}{2}\rho V_\infty^2}$$

< 0 implies cavitation

2.5 Properties Involving the Flow of Heat

For flows involving heat transfer such as gas dynamics additional thermodynamic properties are important, e.g.

specific heats	c_p and c_v	J/kg·°K
specific internal energy	u	J/kg
specific enthalpy	$h = u + p/\rho$	J/kg

2.6 Elasticity (i.e., compressibility)

Increasing/decreasing pressure corresponds to contraction/expansion of a fluid. The amount of deformation is called elasticity.

$$dp = -E_v \frac{dV}{V} \qquad dp > 0 \Rightarrow \frac{dV}{V} < 0$$

$\therefore$ minus sign used

$$E_V = -\frac{dp}{dV/V} = \frac{dp}{d\rho/\rho} = \frac{N}{m^2} \qquad E_V = \rho\frac{dp}{d\rho}$$

Alternate form: $M = \rho V$

$dM = \rho dV + V d\rho = 0$ (by definition)

$$-\frac{dV}{V} = \frac{d\rho}{\rho}$$

Liquids are in general incompressible, e.g.

$E_v = 2.2\ GN/m^2$ water

i.e. $\Delta V = .05\%$ for $\Delta p = 1 MN/m^2$

(G=Giga=10^9 M=Mega=10^6 k=kilo=10^3)

Gases are in general compressible, e.g. for ideal gas at T = constant (isothermal)

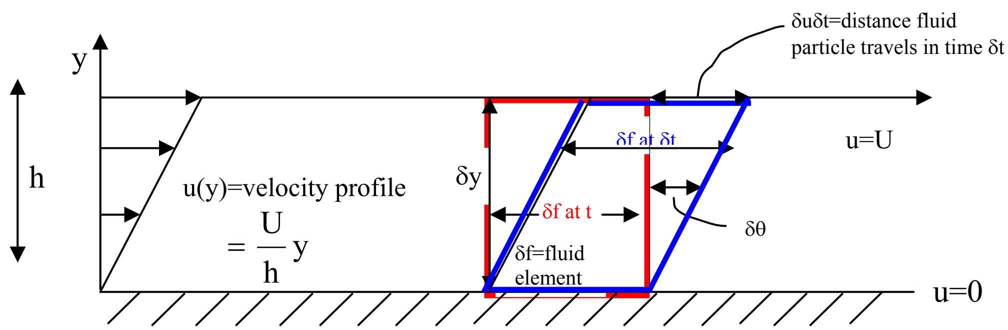
$$\frac{dp}{d\rho} = RT$$

$$\boxed{E_v = \rho RT = p}$$

2.7 Viscosity

Recall definition of a fluid (substance that deforms continuously when subjected to a shear stress) and Newtonian fluid shear / rate-of-strain relationship ($\tau=\mu\dot{\theta}$).

Reconsider flow between fixed and moving parallel plates (Couette flow)



Newtonian fluid: $\tau=\mu\dot{\theta}=\mu\frac{\delta\theta}{\delta t}$

$\tan\delta\theta=\frac{\delta u\delta t}{\delta y}$ or $\delta\theta=\frac{\delta u\delta t}{\delta y}$ for small $\delta\theta$

therefore $\delta\dot{\theta}=\frac{\delta u}{\delta y}$ i.e., $\dot{\theta}=\frac{du}{dy}$ = velocity gradient

and $\tau=\mu\frac{du}{dy}$

Exact solution for Couette flow is a linear velocity profile

$$u(y)=\frac{U}{h}y$$

Note: u(0) = 0 and u(h) = U

i.e., satisfies no-slip boundary condition

$$\tau=\mu\frac{U}{h}=\text{constant}$$

where

U/h = velocity gradient = rate of strain

μ = coefficient of viscosity = proportionality constant for Newtonian fluid

$$\mu=\frac{\tau}{\frac{du}{dy}}=\frac{N/m^2}{\frac{m}{s}/m}=\frac{Ns}{m^2}$$

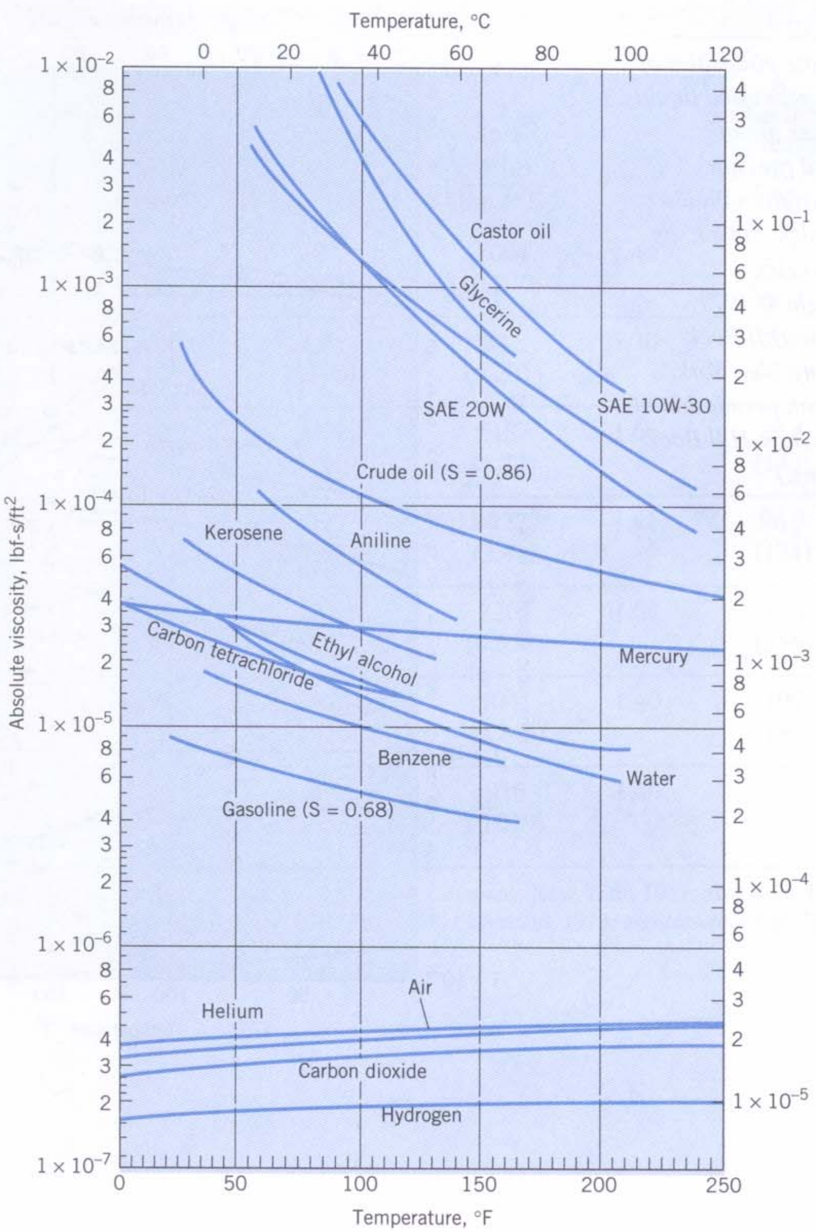
$$\nu=\frac{\mu}{\rho}=\frac{m^2}{s}=\text{kinematic viscosity}$$

μ = μ(fluid;T,p) = μ(liquid;T) = μ(gas/liquid;T)

gas and liquid μ↑ p↑, but smal Δμ

gas: μ↑ T↑
liquid: μ↓ T↑

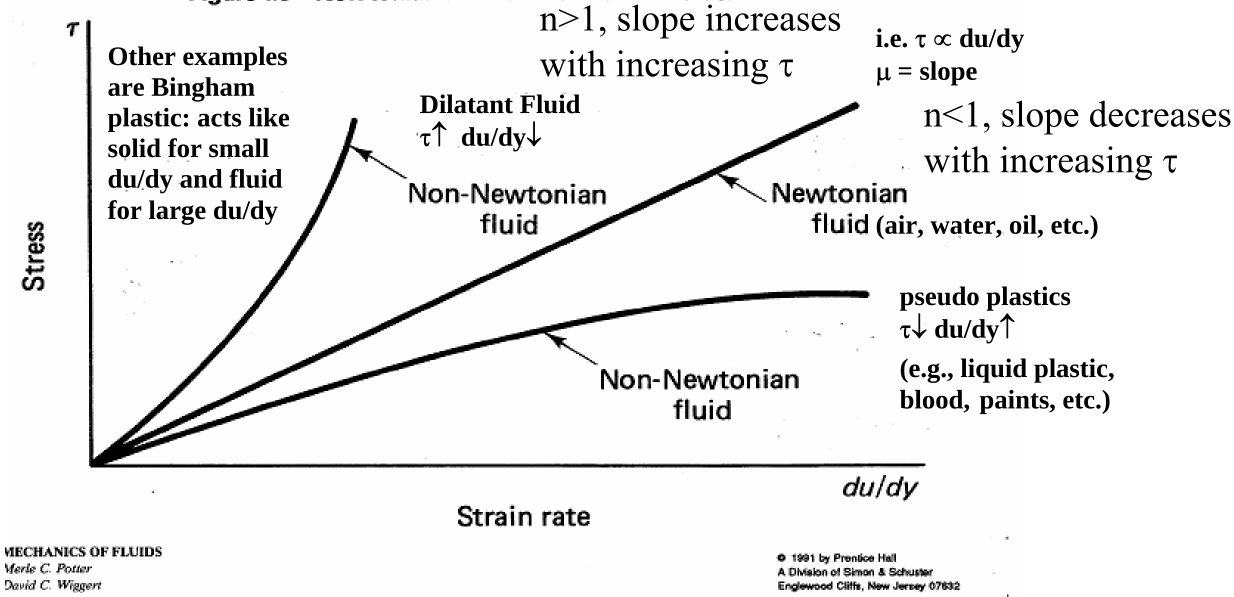
Due to structural differences, more molecular activity, decreased cohesive forces


Temperature, °C
0
20
40
60
80
100
120
Absolute viscosity, lbf-s/ft^2
Absolute viscosity, N-s/m^2
Castor oil
Glycerine
SAE 20W
SAE 10W-30
Crude oil (S = 0.86)
Kerosene
Aniline
Mercury
Carbon tetrachloride
Ethyl alcohol
Benzene
Water
Gasoline (S = 0.68)
Helium
Air
Carbon dioxide
Hydrogen
0
50
100
150
200
250
Temperature, °F

Newtonian vs. Non-Newtonian Fluids

Dilatant: $\tau\uparrow$ dV/dy$\uparrow$
Newtonian: $\tau \propto$ dV/dy
Pseudo plastic: $\tau\downarrow$ dV/dy$\uparrow$

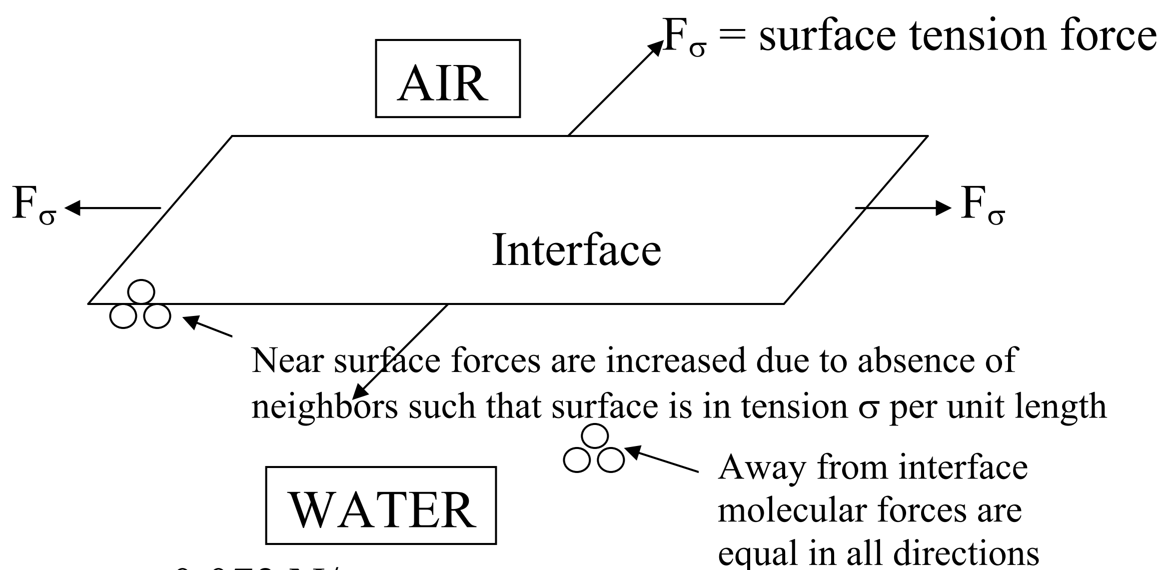
Figure 1.8 Newtonian and non-Newtonian fluids.



$$\tau_{xy} \sim 2k\varepsilon_{xy}^{n}$$

2.8 Surface Tension and Capillary Effects

Two non-mixing fluids (e.g., a liquid and a gas) will form an interface. The molecules below the interface act on each other with forces equal in all directions, whereas the molecules near the surface act on each other with increased forces due to the absence of neighbors. That is, the interface acts like a stretched membrane



$\sigma_{air/water} = 0.073$ N/m

$F_\sigma = \sigma \times L =$ tangent to interface, L=length of cut

Effects of surface tension:

1. Capillary action in small tube $\Delta h = 4\sigma/\gamma d$

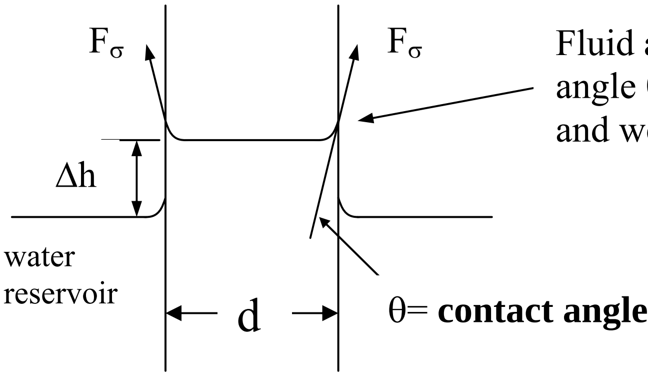
2. Pressure difference across curved interface

   $\Delta p = \sigma/R$ R = radius of curvature

3. Transformation of liquid jet into droplets

4. Binding of wetted granular material such as sand

Example



capillary tube d = 1.6mm = 0.0016m

$F_\sigma = \sigma \times L$, L=length of contact line between fluid & solid

water reservoir at 20° C, σ = 0.073 N/m, γ = 9790 N/m^3

$\Delta h = ?$

$\Sigma F_z = 0$

$F_{\sigma,z} - W = 0$

$\sigma \pi d \cos\theta - \rho g V = 0$ $\theta \sim 0° \Rightarrow \cos\theta = 1$

$\rho g = \gamma$

$$\sigma \pi d - \gamma \Delta h \frac{\pi d^2}{4} = 0$$

$\forall = \Delta h \dfrac{\pi d^2}{4}$ =Volume of fluid above reservoir

$$\Delta h = \frac{4\sigma}{\gamma d} = 18.6 \text{ mm}$$